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THEORETICAL MECHANICS

FLUIDS

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PREFACE.

THIS book is written to meet the requirements of the syllabus of the Science and Art Department for the Elementary Stage of Division II. of Subject VI. Theoretical Mechanics.

I have endeavoured, whilst following out closely the lines of the syllabus, to include every part of the subject which might fairly come within the scope of the syllabus, and yet not to give anything which would be beyond the elementary student.

The book will also be found sufficient for the requirements of the London University Matriculation Examination in the sections of the subject dealt with.

The special feature of the book is the large number of examples which are fully worked out. The reason for including these is that, after a long experience in teaching, I have found nothing so helpful to the student—especially the beginner—as the working out of numerical examples. Examples worked out by the student fix the subject-matter on the mind much more than simply reading over the text. The student cannot be too much impressed with the importance of this branch of his work. To make the work complete in this

direction, a very large number of examples will be found at the end of each chapter. For these I am indebted to Mr. G. R. Vine (Int. Sc.), Head Master of the Hunter's Bar Board School, Sheffield, formerly teacher of Mathematics and Theoretical Mechanics in the Central Science Schools, Sheffield.

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CHAPTER I.

MOTION.

WHEREVER we look we see bodies in motion, *i.e.* bodies which are constantly changing their position; such are the pendulum of the clock as it swings to and fro, the cab in the street, the football of the boys in the playground. Yet a little thought will convince us of the fact that these same bodies are powerless to set themselves in motion; *e.g.* the stone by the roadside will remain there until it is set in motion by some power outside itself, the football remains at the starting-point until the starter gives it a blow. Again, notice the difference between the motion of a stone thrown along the road pavement and that of another thrown along smooth ice; we see that the motion of the one ceases very much sooner than that of the other, and we also see that the smoother the ice the further the stone travels, and we can very safely conclude that if everything opposing the stone's motion were removed, the stone would still continue to be in motion, and that the body, once set in motion, has no power of itself to cease moving; or, in other words, that its motion ceases, not because it influences itself to do so, but because it is acted on in some way or other by some body outside itself.

Again, a little more consideration will show us that a body is unable of itself to change the *direction* of its motion. If, in the experiment with the stone and the smooth ice, we replace the rough stone by, say, a smooth cricket ball, and no object be allowed to oppose its course, the cricket ball will not only continue moving, but will do so in the direction in which it was first started, and will not turn either to the right or to the left.

It may be said that the stone thrown vertically upwards turns of itself and comes down in the opposite direction; but we shall see later on that this turning is not brought about by itself, but by some external cause.

We have, then, to discuss the different states of bodies—*motion* and *rest*—with their causes and the causes of the changes which take place in both conditions.

Let us first, then, clearly understand the two terms *motion*, *rest*.

MOTION may be defined as *change of position*; the pendulum, cab, football, etc., referred to, occupy different positions at successive instants of time—they are in *motion*.

REST may be defined as *permanence of position*; the ink-stand, book, desk, map, and many other bodies in the room preserve the same positions at successive instants of time—they are at *rest*.

We must examine these terms a little more closely.

(i.) *Motion*. Two trains are moving on parallel lines of rails, the one, A, say, at 30 miles an hour, the other, B, say, at 40 miles an hour; if we consider the motion of each train separately, we get 30 miles and 40 miles an hour respectively, whereas if we consider the motion of A with regard to the motion of B, we get rates of 70 miles and 10 miles an hour, according as the trains are moving in opposite or the same directions.

Thus we have *relative* and *absolute* motion. The *relative* motion of a body is its change of position with regard to surrounding objects; thus a person may be at *rest* with regard to the omnibus in which he is riding, but in *motion* relative to the houses, shops, trees, etc., past which the omnibus runs.

Example of Relative Motion.—Two men—A and B—start in a race together. A runs at the rate of $\frac{3}{4}$ mile per minute, B at the rate of $\frac{7}{8}$ mile per minute. What is B's motion relative to that of A's? Give the answer in yards per minute.

A runs at the rate of $\frac{3}{4}$ mile per minute, or $\frac{3 \times 1760}{4}$ yards per minute; *i.e.* at 1320 yards per minute.

B runs at the rate of $\frac{7}{8}$ mile per minute, or $\frac{7 \times 1760}{8}$ yards per minute; *i.e.* at 1540 yards per minute.

Therefore B's motion relative to that of A's is $(1540 - 1320)$ yards per minute, or 220 yards per minute.

The *absolute* motion of a body is its change of position when spoken of with regard to certain fixed points, lines, or planes in space.

(ii.) *Rest* may also be spoken of as *relative* or *absolute*. By *relative* rest we mean that the body keeps the same position with regard to surrounding bodies. Thus the seat on which I

sit is at rest with regard to the desks, books, walls, etc., of the room; but remembering that, because the earth itself is in motion, these seats, books, walls, etc., must be moving too, we at once see that every object with which we are acquainted, even including the earth, moon, sun, and other celestial bodies making up our solar system, is in constant relative motion.

The *absolute* rest of a body is its permanence of position with regard to certain fixed points, lines, or planes in space, and as every object with which we are acquainted is in relative motion, we shall confine our attention to relative motion and relative rest.

We have to determine methods by which the motion of bodies may be measured, and to do this we must know the *rates of motion* of the bodies considered; that is, we must know their *velocity*.

The *velocity* or *rate of motion* of a body is found by observing the amount of space passed over in a given time.

A train travels over 40 miles in an hour, its velocity is said to be 40 miles an hour; a man is able to walk 100 yards in 1 minute, his velocity is said to be 100 yards per minute; a stone falls through 32 feet in 1 second, its velocity is said to be 32 feet per second.

It will at once be seen that the velocities will be expressed by different numbers, according as different units of time and space are used; *e.g.* a velocity of 15 miles an hour is the same as a velocity of $\frac{15 \times 1760}{60}$ yards per minute; *i.e.* the same as a velocity of $\frac{15 \times 1760 \times 3}{60 \times 60}$ feet per second; *i.e.* a velocity of 15 miles an hour is equal to a velocity of 22 feet per second.

Examples on Change of Units.—(i.) A stone, after falling from a height of 144 feet, reaches the ground with a velocity of 96 feet per second. How many miles an hour is this equivalent to?

A velocity of 96 feet per sec. $= \frac{96}{3}$ yards per sec. $= \frac{96}{3 \times 1760}$ mile per sec. $= \frac{96 \times 60}{3 \times 1760}$ miles per min. $= \frac{96 \times 60 \times 6\cancel{0}}{3 \times 176\cancel{0}}$ miles per hour $= \frac{12 \times 60 \times \cancel{2}}{\cancel{22}}$ miles per hour $= \frac{12 \times 60}{11}$ miles per hour $= \frac{720}{11}$ miles per hour $= 65\frac{5}{11}$ miles per hour.

(ii.) A bird flies 77 miles in 2 hours 20 minutes; supposing its speed to have been uniform, what is the velocity in feet per second?

A velocity of 77 miles in 2 hours 20 mins. = a velocity of $77 \times 1760 \times 3$ feet in 2 hours 20 min. = a velocity of $\frac{77 \times 1760 \times 3}{140}$ feet per min. = a velocity of $\frac{\overset{11}{\cancel{77}} \times 176\cancel{0} \times \cancel{3}}{\underset{20}{\cancel{140}} \times \underset{2}{\cancel{60}}}$ feet per sec. $= \frac{11 \times \overset{22}{\cancel{176}}}{\underset{5}{\cancel{20}} \times \cancel{2}}$

$= \frac{242}{5}$ feet per sec. $= 48\frac{2}{5}$ feet per sec.

The unit of time used everywhere is one second; in England we use one foot as the unit of length.

If the velocity of the body be always the same; *i.e.* if the same number of feet be passed over in each successive small interval of time, or, generally, if equal spaces be always passed over in equal times, the velocity is said to be *uniform* and the motion of the body is *uniform.*

On the contrary, if unequal spaces be passed over in equal times we have *variable* velocity, and therefore *variable* motion.

To measure the space passed over in a given time is an easy matter if we know the velocity, the velocity being uniform.

Let v represent the number of feet passed over in 1 second.

Then $2v$ will represent the number of feet passed over in 2 seconds.

$3v$ will represent the number of feet passed over in 3 seconds.

$4v$ will represent the number of feet passed over in 4 seconds.

And generally if t represent the number of seconds of time, the space passed over must be represented by tv.

That is, space passed over = velocity × time, or using the initials of the words, we get the equation:

$$s = vt$$

But the cases most generally met with fall under the head of variable motion caused by variable velocity.

To illustrate. We may notice the motion of the railway train. We say it is going at the rate of 40 miles an hour; by this we mean that if it were to travel at this same rate during one hour it would pass over a distance of 40 miles, and at

first sight we might suppose its motion to be uniform, *i.e.* that during each second of the hour exactly the same length is passed over as in every other second; but we know that this is not the case, for the velocity is constantly changing according as the train is leaving or entering the station or whether its path is uphill or downhill. This is therefore a case of variable motion.

Again, we know that if a stone be thrown into the air, the first few feet of its path are passed over much quicker than the next few, and after a short time it comes to rest and then descends; and as it descends its velocity gradually increases: this is therefore another case of variable velocity.

How, then, shall we measure variable velocity? This is done by observing the space through which the body moves during a short interval of time, then by calculating on the supposition that, during its motion, it moves with the velocity it had at the particular moment of observation.

We might also explain what is meant by *mean* velocity. This can best be done by an example. A man walks for 11 hours; during the first 3 hours he walks 10 miles, during the 4th, 6th and last he walks $2\frac{1}{3}$ miles an hour; during the rest of the time he travels over 16 miles. Altogether he travels over $10 + 7 + 16$ miles. His mean velocity or his average velocity $= \frac{10 + 7 + 16}{11}$ miles an hour $= \frac{33}{11}$ miles an hour $= 3$ miles an hour.

The *mean* velocity of a moving body is thus found by dividing the total space passed over in a given time by the given time.

We will now state NEWTON'S FIRST LAW OF MOTION.—*No body has any power of itself to change its state, whether of rest or of uniform motion; or, in other words, if a body be in a state of rest or of motion some power outside itself must be impressed upon it to cause it to change its state; or, again, if a body be at rest it will continue in that state unless acted upon by some external cause, or, if it be in motion, it will continue to move in a straight line with uniform velocity unless acted upon by some external cause.*

This property of matter is sometimes briefly expressed by saying that matter possesses the property of *inertia*.

Numerous examples of this First Law of Motion will occur at once to the mind of the student.

EXAMPLES ON CHAPTER I.

1. What is meant by the word "velocity"? Point out the difference between motion and velocity. Correct "rate of velocity."

2. What units of time must be employed in specifying a "velocity"? What units are commonly employed in dynamical questions for time and distance? [*S. & A.*

3. What do you understand by "foot-sec. units," "mile-hour units," "yard-minute units," "kilometre-hour units"?

4. What is meant by uniform or constant velocity? If a distance of 45 feet is passed over in 5 secs., the next 63 in 7 secs., and the next 108 in 12 sec., is this body probably moving with uniform velocity? Give reasons for your answer. Why "probably"? What are the important words in the definition of "constant velocity"?

5. A velocity is expressed by 15 foot-sec. units, what will it be in yard-min., and mile-hour units?

6. A train is said to be moving at the rate of 45 miles per hour. Say clearly what this means. How far would it move in 1 sec. taking the unit of distance as 1 foot?

7. How is the velocity of a point measured when uniform? How when variable? The velocity of a body is 24 if measured in feet and seconds. What will it be if measured in miles and hours?

8. How would you compare a number of velocities? Compare the following velocities: 16 foot-sec. units; 45 miles in 3 hours; and 20 yards per minute.

9. A man walks with uniform speed for 3 hours, during which time he covers 14 miles; find his velocity in foot-sec. units and metre-sec. units (metre = 39 ins.).

10. What is meant by the relative velocity of two bodies. If Tom walks towards Harry at 3 miles per hour, and Harry towards Tom at $3\frac{1}{4}$ miles per hour, what is their relative velocity, and how long would they be in meeting on a straight road if they were 26 miles apart at the start?

11. There are two trains on parallel lines, 2 miles apart—engines in front, vans behind. First train is 220 feet long, and second is 330. The first train moves towards the second at the rate of 45 miles per hour, and the second towards the first at the rate of 50 miles per hour. What time will elapse before they pass each other?

12. Find the average velocity of the following in foot-sec. units: 3 miles in 20 min., 65 ft. in 13 secs., 18 yds. in 2 min., 40 miles in 1 hour, and 19 ft. in 3 min.

13. Two trains—A and B—moving towards each other on parallel rails uniformly at the rate of 30 miles and 45 miles an hour respectively, are five miles apart at a given instant: how far apart will they be at the end of 6 mins. from that instant, and what distance are they from the first position of A? [*S. & A.*

CHAPTER II.

FORCE.

So far, in speaking of bodies in motion, we have spoken of them as being influenced by bodies external to themselves, and our language has been somewhat indefinite. We now come to the causes of the changes of states of rest and uniform motion of bodies, and to these causes we give the name of *force.*

FORCE, then, *is that which changes or tends to change the state of a body, whether of rest or of uniform motion.* The windmill's sails are set in motion by the force of the wind; the water-wheel is itself started by the force of the water in its fall from the dam; the engine of the train is driven by the force of the steam; the shot from the boy's catapult is projected by the force of the spring; and so on with numberless examples.

Again, when there is a change in the direction of the motion, we see that this is caused by the application of some force external to the body. The stone thrown vertically upwards is constantly acted on by the force of gravity; therefore its velocity constantly decreases until it comes to rest, and then, as it is still acted on by the same force, it begins to descend. The boy ties a stone to the end of a string, and then whirls it round at arm's length; the stone is made to move in the circle by the force of the tension of the string.

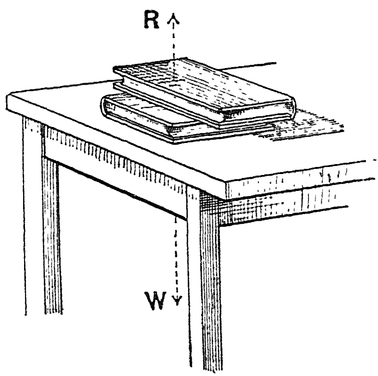

FIG 1.

Again, bodies are kept at rest by the action of two or more forces upon them; the map as it hangs on the wall is pulled downwards by the force of gravity, but kept in its place by the force of the nail brought to bear on it by the tension of the cord suspending it.

The books lying on the table would fall to the earth, but are prevented from doing so by the opposing force of the table.

Such forces as those of the nail and the table are called *reactions* or *resistances*.

We shall come across many such instances of bodies at rest acted on by forces. In such cases we may safely say that one or more of the forces balance the remainder, and in all such cases the forces are said to be *in equilibrium*.

Examples of bodies acted upon by forces in equilibrium.

1. The ladder as it rests against the wall acted on by its own weight and the resistance and friction of the wall and the ground.

2. The block of stone suspended from the beam by means of the pulley.

Here the forces are its own weight and the tension of the string.

3. The ordinary balance used in the grocer's shop.

I hold a book in my hand; I feel a force pulling the book to the earth; the force is the attraction of the earth upon the book. To keep the book at rest, I must exert exactly that same amount of force in the opposite direction. This amount of force with which the earth attracts the book we call the *weight* of the book.

We must be careful not to confuse the term *weight* with the term *mass*. As we have said, *weight* is the amount of the force of the attraction of the earth upon the given body. The *mass* of the body is the amount of *matter* in the body.

By *matter* we understand that which we can see, touch, smell, etc.

Matter may be solid, liquid, or gas.

Solids have a definite size and shape, as stone, ice, wood, iron, etc.

Liquids have a definite size, but not a definite shape, as water, mercury, oil, etc.

Gases have neither definite size nor shape, as air, oxygen, steam, etc.

Water is known in the three forms, as ice, water, steam.

Whilst mass is always measured by weight, yet the two terms must be kept distinct, the weight being the amount of force which the attraction of the earth exerts on the mass.

If g represents this attraction—

W	weight of the body,
m	mass ,, ,,
We have . .	$W = mg$.

We must, again, distinguish carefully between the two pre-

ceding terms and the term *density*, which is the relation between the amount of matter of a body and the space occupied by the body.

E.g. A pound of lead will contain as much matter as a pound of cork, but we know that there will be a great difference between the spaces occupied. Again, a sovereign will weigh much more than the same bulk of blotting-paper, simply because the particles of gold making up the sovereign are packed closer together than the particles of the blotting paper; or, in other words, one is *denser* than the other.

$$\text{Thus density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{or } D = \frac{m}{V}$$

Specific Gravity.—We have just said that the matter contained in different bodies is not to be compared by their bulk, or by the amount of space occupied. How shall we compare them? We know the weight of a certain quantity, say a cubic foot of water. If we can obtain the weight of a cubic foot, say, of copper, we can then at once compare the two weights, or we can compare the amounts of matter contained in the two bodies. *E.g.* we know that—

a cubic foot of water weighs 1000 ounces,
" " copper " 8900 "

These figures tell us that a certain bulk of copper contains 8·9 times more matter than the same bulk of water.

The specific gravity of a body is its relative weight compared with water.

To put this in a form easily to be remembered, and to assist calculation, let S be the specific gravity of a substance, V its volume, w the weight of a certain volume of water chosen as a unit. Then if W be the weight of the substance, wV will be the weight of the same volume of water, and therefore, by the definition given—

$$\frac{W}{wV} = S$$

The specific gravity of a *gas* is usually taken as its relative weight compared with air, *i.e.* the relative density of a gas is the ratio of the weight of a certain volume of the gas to the weight of the same volume of air under the same conditions.

The subject of specific gravity will be more fully considered later on.

Example 1.—A piece of metal of specific gravity 9·25 weighs 185 oz. Find its volume.

$$\text{Using the formula } S = \frac{W}{wV}$$

$$9{\cdot}25 = \frac{185}{1000 \times V}$$

w = weight of a cubic foot of water = 1000 ozs.

$$\therefore 1000\,V = \frac{185}{9{\cdot}25} = 20$$

$$\therefore V = \frac{20}{1000} = \frac{1}{50}$$

$$\text{Ans. } \frac{1}{50} \text{ cub. ft.}$$

Example 2.—The specific gravity of mercury is 13·6; find the length of a column of water 1 inch in diameter which shall be equal in weight to a column of mercury of the same diameter which is 30 inches in length.

$$\text{Length of column of water} = 13{\cdot}6 \times 30 \text{ ins.}$$

$$= \frac{13{\cdot}6 \times 39}{12} \text{ ft.}$$

$$= 34 \text{ ft.}$$

To determine the effects of a force upon a body we must know at what point of the body the force is applied, the magnitude of the force, and the direction of the force; or, briefly, to "know" a force, we must have determined—

(i.) Its point of application.
(ii.) Its magnitude.
(iii.) Its direction.

Point of Application.—Up to the present we have spoken of forces acting upon *bodies*; it would be more correct to speak of each force acting at a particular *point* of the body. This point is called the *point of application* of the force.

As an illustration of the importance of the point of application of the force we may use the case of the boy's "see-saw." Every boy knows that the further off the block in the middle he sits, the greater effect he will have; or, in other words, if he shifts the point of application of his weight further towards the end of the beam, the greater power he has.

Another case is found in the point of suspension of the movable weight of the common steelyard.

Here might also be mentioned the principle that a force may be applied at any point in the line of its direction, remembering that there must be a rigid connection with the first point of application.

Thus a weight of 4 lbs. weighs just as much whether I hold

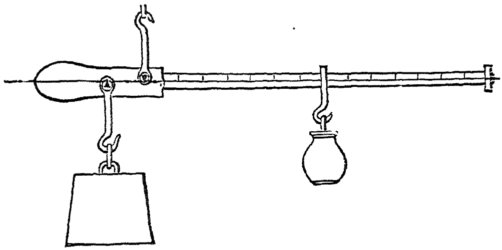

FIG. 2.

it by a length of one inch, or by a length of two feet, or by any length whatever. The pointer in the spring balance is not affected by the distance between the weight and the hook, but by the weight.

Magnitude.—As we have found means by which to measure time by days, hours, seconds, etc., and space by yards, feet, inches, etc., so we must choose some unit by which we can measure force.

We have already defined the term "weight," and by saying a body weighs one pound we mean that the attraction of the earth on a pound of matter is a force of one pound. We can use this amount of force as a unit. It is called the *gravitation unit.*

Or if we define the unit of force as that amount of force which acting for one second on one pound of matter will produce a velocity of one foot per second, we shall obtain a different figure. To this unit we give the name *absolute unit of force.*

We find by experiment that if a body weighing one pound be allowed to fall freely from rest for one second it will have at the end of that second a velocity of 32 feet per second (roughly), *i.e.* the force represented by the weight of one pound can impart to the falling body 32 times the velocity which the unit of force would do; therefore the pound weight must be 32 times the unit of force, or, in other words, we find this unit

is equal to $\frac{1}{32}$ lb., or $\frac{1}{2}$ oz. To this unit thus found is given the name *British absolute unit of force*, or the *poundal.* Therefore the gravitation unit equals 32 British absolute units, or 32 poundals.

Examples of Change of Units.—(i.) How many poundals of force are there in 7, $16\frac{2}{3}$, 96 gravitation units? and how many gravitation units in forces of 48, 256, 793 British absolute units?

From what has been said, we take the gravitation unit to be equal to 32 British absolute units or poundals.

$\therefore$ 7 grav. units $= 7 \times 32$ British absolute units or poundals $= 224$ poundals.

$16\frac{2}{3}$ " $= 16\frac{2}{3} \times 32$ " " $= 533\frac{1}{3}$ "

96 " $= 96 \times 32$ " " $= 3072$ "

$\therefore$ 48 British absolute units or poundals $= \frac{48}{32} = 1\frac{1}{2}$ grav. units.

256 " " " $= \frac{256}{32} = 8$ "

793 " " " $= \frac{793}{32} = 24\frac{25}{32}$ "

Some idea of the magnitude of the poundal may be experimentally found thus. Tie a half-ounce weight to the end of a string; the force needed to hold up this weight is a little more than the poundal.

The magnitude of a force *is the number of units of force it is equal to.* Thus a force of 5 units is half the magnitude of one of 10 units.

Direction.—The direction of a force *is the straight line in which the body would move if the force were the only one acting upon it.*

It will at once be seen that in all these respects, viz. point of application, magnitude, and direction forces can be represented by straight lines.

Suppose a force F of F units acts upon a point A. This

FIG. 3.

point begins to move in some definite direction, A B, according to the direction of the force F.

If we choose some unit of length to represent the unit of force, we shall be able to make the straight line A B the same number of units of length as there are units of force in the force F.

The arrow-head will show sufficiently the direction of the

force F, or we can speak of its direction as AB, or BA, according as it moves from A to B, or from B to A.

Example.—A force of 10 lbs. can be represented by 10 inches, $2\frac{1}{2}$ inches, or 10 feet, according as we choose 1 inch, $\frac{1}{4}$ inch, or 1 foot to represent 1 lb. What length we use as our unit of length can be decided by convenience for working.

Examples of Use of Units of Length for Magnitude of Force.—(i.) If a line 1 inch long represent a force of 1 lb., what would be the lengths of lines representing 1 cwt., 5 ozs., 45 lbs. respectively?

A force of	1 lb.	is represented	by a line	1 in. long.
,,	112 ,,	,,	,,	112 ,,
,,	112 ,,	,,	,,	$\frac{112}{12}$ yds. lg. = $9\frac{1}{3}$ yds. lg.
,,	5 ozs.	,,	,,	$\frac{5}{16}$ in. long.
,,	45 lbs.	,,	,,	$\frac{45}{12}$,,
,,	45 ,,	,,	,,	$3\frac{3}{4}$,,

(ii.) A force of 128 poundals is represented by a line 4 feet long; what unit of length represents the gravitation unit?

A force of	128 poundals	is represented	by a line	4 ft. long.
,,	1 ,,	,,	,,	$\frac{4}{128}$,,
,,	32 ,, or 1 grav. unit	,,	,,	$\frac{4 \times 32}{128}$,,
,,	1 grav. unit	,,	,,	1 ,,

There are very few instances, if any, of a body being acted on by a single force, so that we have to consider the joint action of two or more forces on the same body, and it will help us much if we can reduce any or all of the forces thus acting to one force. This one force is called the *resultant* of the others; *i.e.* the *resultant of two or more forces acting upon a body is that* ONE *force which will produce the same effect both in magnitude and direction as the several forces combined.*

The forces thus combined are called the *components* of the resultant force. Thus *component* and *resultant* are relative terms. Illustrations—

(i.) Two weights of 1 oz. each in the pan of a balance will produce the same effect as one of 2 ozs.

(ii.) Two men kicking a football at the same instant may be replaced by one man who could produce exactly the same effect on the ball as the two combined.

The simplest cases of resultant are obtained from

(i.) Forces acting in the same direction and on the same point.

E.g. Forces of 8 lbs., 3 lbs, and 5 lbs. acting on the same

point in the same direction may be replaced by a single force of $(8 + 3 + 5)$ lbs., or one of 16 lbs.; or generally if forces P, Q, S act on the same point in the same direction they can be replaced by one force R, where $R = P + Q + S$.

FIG. 4.

(ii.) Forces acting in the same straight line on the same point, but not all in the same direction.

Here we find the resultant of the forces acting in one direction, and the resultant of those acting in the opposite direction. The final resultant is then found by taking the difference of the two resultants thus found, its direction being in the direction of the greater resultant of the two.

E.g. Forces of 3 lbs. and 6 lbs. act on a point in one direction, and forces of 3 lbs. and 5 lbs. in the opposite direction. These are equal to two forces, one of 9 lbs., the other of 8 lbs. acting in opposite directions; the final resultant is one of 1 lb. acting in the direction of the resultant of 9 lbs.

FIG. 5.

$$\begin{aligned} \text{Generally } R &= R_1 - R_2 \\ \text{where } R_1 &= P + Q + S + \ldots \\ R_2 &= T + V + W + \ldots \end{aligned}$$

In this case, if $R =$ zero the forces are in equilibrium.

EXAMPLES ON CHAPTER II.

1. From what well-known statement do we get a definition of force? Give a definition in your own words, and compare it with the definition alluded to above.

2. Can we have force without matter? Can we have mass without force?

3. Mention the items that must be stated in order that a force may be considered to be known. Show that a force may be represented by a straight line. [*S. & A.*

4. What is a resultant force? and what is a component force?

5. What is the difference between weight and mass? What are the two meanings of "weight"? Define weight.

6. What is an absolute unit of force? What is the British absolute unit? What do you mean by "absolute"? [*S. & A.*

7. What is a gravitation unit? What is the British gravitation unit? What is meant by gravitation? [*S. & A.*

8. Convert 19 British gravitation units to British absolute units ($g = 32$) and 160 poundals to pounds.

9. How do we usually compare the masses of two bodies?

10. What is density? What is specific gravity? Explain the letters in M = V D.

11. How is density measured? What is meant when a rod and a lamina is said to be of uniform density? [*S. & A.*

12. A body A has a volume of 1·35 cub. ft., and a specific gravity of 4·4. Another body B has a volume of 10·8 cub. ins., and a specific gravity of 19·8. What is the ratio of the mass of A to that of B. (Note M = V D.) [*S. & A.*

13. In a certain state of the atmosphere, 100 cub. ins. of air weigh 31 grs. At the same temperature 30 cub. ins. of mercury weigh 14·88 lbs. Find the number of cubic inches of air which contain as much matter as a cubic inch of mercury. Would this number be constant all the year round in the same place? If not, why not? [*S. & A.*

14. How are the densities of two bodies compared? If 5 cub. ins. of mercury weigh 2·45 lbs., and 2 cub. ins. of cast iron weigh ·52 lbs., what ratio does the density of mercury bear to that of cast iron? [*S. & A.*

15. What is the specific gravity or specific density of a solid? A circular piece of gold and a common cork have equal weights and diameters, the cork is $1\frac{3}{4}$ ins. long; how thick is the piece of gold, the specific gravity of gold being 19·25 and that of cork ·25? [*S. & A.*

16. If the specific gravity of brass be taken as 8·4, find the weight of a bar of brass, 10 ins. long and 4 sq. ins. in section.

17. Find the weight of a piece of oak 7 ft. high, 3 ft. wide, and $1\frac{1}{2}$ ins. thick—taking the specific gravity of oak as ·93.

18. Find the weight of a block of lead (specific gravity 11·35) 6 ft. long, 2 ft. wide, and 2 ins. thick.

19. Find the resultant of the following: force of 3 lbs. to the right, 64 poundals to the left; 5 British gravitation units to left, and 160 British absolute units to the right.

20. If a line 3 ins. long represents 4 lbs. what line will represent 96 poundals.

21. If forces of 5, 8, 13, act on a point and maintain equilibrium, show in a diagram how they must act.

22. What force will make the following set, one in equilibrium?

$$-18 - 2 + 19 + 17 - 4 + 5$$

23. Explain the meaning of each letter in $W = mg$. What is the ordinary unit of mass?

24. Of two bodies one has a volume of 5 cub. ins., and the other $\frac{1}{2}$ cub. ft. The former weighs 15 ozs., the latter 12·8 lbs. What is the ratio of the mass of the first to that of the second? What is the ratio of their densities?

25. Two bodies are 9 and 12 cub. ins. in volume respectively, their weights are 12 and 9 ozs. respectively. What is the ratio of their densities?

26. How is quantity of matter ascertained? If a cubic inch of water, contained exactly a gramme of matter, what would be the quantity of matter —estimated in grammes—in a cubic foot of lead? A linear foot being 30·45 centimetres, and the specific gravity of lead being 11·445. [*S. & A.*

27. If 100 cub. ins. of oxygen (under certain circumstances of pressure and temperature) weigh 35 grains, and a cubic inch of mercury weighs ·49 lbs., how many cubic inches of oxygen contain the same quantity of matter as a cubic inch of mercury? [*S. & A.*

CHAPTER III.

PARALLELOGRAM OF FORCES.

THE most important case of finding the resultant of forces acting upon a point is when a number of forces are thus acting at angles with each other.

First, let there be two such forces.

The resultant is then found by the principle called the Parallelogram of Forces.

We can best arrive at this experimentally by the aid of a simple piece of apparatus.

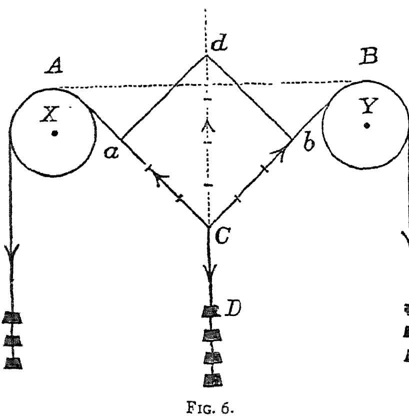

FIG. 6.

Take two small pulleys, X, Y, which can move without friction; fix them in the blackboard. Over these pulleys pass a fine thread. To each end of the thread, A, B, affix certain chosen weights. In the first case let the weight at A be equal to the weight at B, and be, say, 3 ozs. At some other point, C, in the thread between X and Y affix a third weight, say 4 ozs. The point C will descend until the whole system is at rest.

We have now the point C acted on by three forces, viz. the tensions in the strings C A, C B due to the weights fastened to A and B, and the tension on the string C D due to the weight fastened at C.

Let us examine our result. To do this, mark on C A and C B three units of length (say three inches). Complete the parallelogram C *a d b*. Join C *d*. We now find that there are three special facts in connection with the line C *d*.

(1) It is in the same straight line with D C.

(2) It bisects the angle A C B.

(3) If measured it will be found to contain as many units of length (inches) as we have ounces in the weight fastened to C; *i.e.* it will be 4 inches long.

The reason for fact (2) is that the weights attached to A and B are equal, as we might have supposed, for there is no reason why C *d* should incline to one force more than to the other. We know that if two boys kick at a football with equal strength at the same moment, but at an angle with each other, the ball must travel midway between the two directions in which the ball is kicked.

Before referring to facts (1) and (3) we will take a more general case.

Use the same apparatus, but here make the weights attached to A and B unequal, say 3 ozs. at A and 4 ozs. at B. If we attach 5 ozs. at C, our result will be similar to the other case, viz. the weights will come to rest. The point C is in equilibrium acted on by three forces.

On C A mark 3 units. On C B mark 4 units, and complete parallelogram and join C *d* as before.

Here facts (1) and (3) agree with (1) and (3) of the last case, but (2) differs, for here we find the angle A C B is not bisected, the angle A C *d* being less than the angle B C *d*.

The facts (1) and (3) teach us that if the lines C *a*, C *b*, represent forces at A and B, then line C *d* represents in magnitude and direction their resultant, for it is in the same straight line with D C, and of the same length as the number of units of force attached to the point C; in other words, the line C *d* represents a force equal and opposite to the force acting on C D, which force balances the combined effect of the forces in C *b* and C *a*.

We must here carefully distinguish between the two forces in C *d* and C D. The one in C *d* represents the *resultant* of the two in C *a* and C *b*, whilst the other in C D *equilibriates* or

balances the two in C a and C b; *i.e.* it is the *equilibriant* of those in C a and C b.

We can now state the PARALLELOGRAM OF FORCES. It is the following. *If two forces acting on a point be represented in magnitude and direction by two straight lines drawn from the point, and a parallelogram be described on these lines as adjacent sides, then the resultant of the two forces will be represented in magnitude and direction by that diagonal of the parallelogram which passes through the point.*

Using this principle, we can find the resultant of two forces in every case by constructing the proper parallelogram and then by measuring the diagonal, remembering that we must choose a convenient unit of length to represent a certain unit of force.

Example.—Find the resultant of forces of 6 lbs. and 4 lbs. acting at an angle of 80°.

Take as unit of length, say, a quarter of an inch to represent a force of 1 lb.

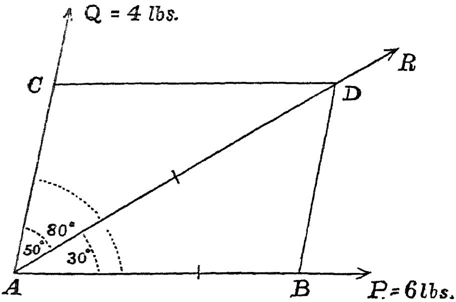

FIG. 7.

Draw line A B; on A B set off $1\frac{1}{2}$ inch; this will represent the force of 6 lbs.; at point A in line A B draw A C, making an angle of 80° (by protractor), on A C set off 1 inch to represent force of 4 lbs.

Complete parallelogram; measure diagonal A D. We find A D by measurement to be 2 inches.

Measure angles C A D, B A D, which are respectively 50° and 30°.

Therefore the required resultant is 8 lbs.
making with force of 6 lbs. an angle of 30°
" " 4 lbs. " 50°

RESOLUTION OF FORCES.—This is the converse of finding the resultant of a number of forces.

To explain. The resultant of a number of forces acting upon a body is that *one* force which will produce the same effect both in magnitude and direction as the several forces combined.

So far we have been finding this *one* force, having given the component forces.

We have now to find the forces which compose or make up this one force, *i.e.* we have to find the components of the resultant.

As any given resultant may be the one force producing the same effect as any number of forces, unless their magnitude and direction are stated, we have to solve a problem which can have any number of solutions, *e.g.* we have a force represented in magnitude and direction by the line AB: this may be the resultant of almost any number of forces, each of almost any magnitude acting in any direction.

We will only examine two cases.

1. When the two components to be found are equal in magnitude. Since they are equal in magnitude their resultant

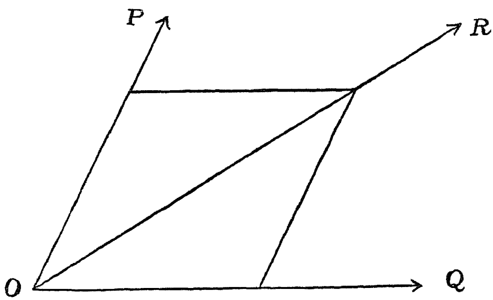

FIG. 8.

will bisect the angle between them. The problem we have to solve then is to find the forces represented in magnitude and direction by the adjacent sides of a parallelogram whose sides are equal, the given force lying between the required forces.

2. When there are two components required at right angles to each other. In this case the required forces are called the *rectangular components* of the forces. The problem is solved by taking the given force as being represented by the

diagonal of a rectangle of which the two required forces are represented by the adjacent sides lying on either side of the given force.

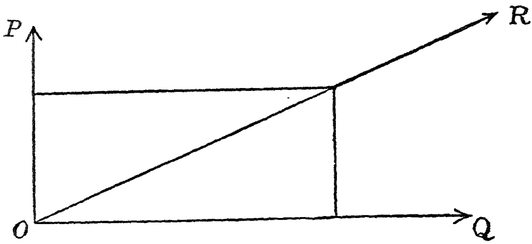

FIG. 9.

This will be better understood by the help of an example.

Example 1.—A force of 50 units acts along a line inclined at an angle of 30° to the horizon. Find by construction (or otherwise) its horizontal and vertical components. [*S. & A.*, 1884.

Take one inch to represent 25 units of force. Draw line A B of any length. At point A in line A B draw line A C at angle of 30° with line A B; make this line 2 inches in length; then line A C represents in magnitude and direction the force of 50 units. Also at point A in line A B draw A D at right angles to A B.

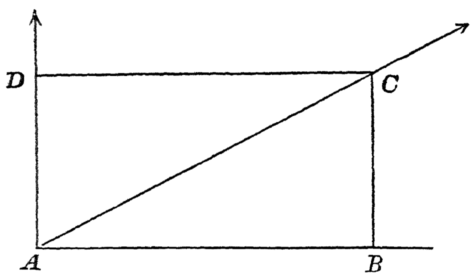

FIG. 10.

Through point C draw line C B parallel to A D, and line C D parallel to A B.

Then line A D represents the required vertical component

in magnitude and direction, and line A B represents the required horizontal component in magnitude and direction.

AD is 1 inch long;

∴ required vertical component is one of 25 units;

AB is $\sqrt{3}$ inches long;

∴ required horizontal component is one of $25\sqrt{3}$ units.

Example 2.—Define the rectangular components of a force. Draw the straight lines, O A, O B, containing a right angle at O; within the right angle draw O P, such that A O P is an angle of $35°$; a force of 18 units acts from O to P; find by construction its rectangular components along O A and O B. [*S. & A.*, 1892.

Rectangular components have already been explained. Draw O A and O B at right angles to each other. By protractor make angle A O P = 35°. Draw O P, 3 inches in length, 1 inch representing 6 units. Through P draw P A parallel to O B, and P B parallel to O A. We have now the rectangular components of O P.

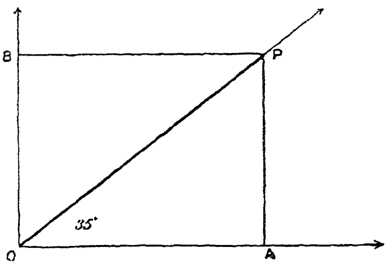

FIG. 11.

Measure O A = $2\frac{1}{3}$ inches.

∴ horizontal component of O P = $2\frac{1}{3} \times 6$ units = 14 units.

Measure O B = $1\frac{7}{8}$ inches.

∴ vertical component of O P = $1\frac{7}{8} \times 6$ units = $11\frac{1}{4}$ units.

The figures obtained in this manner can only be approximations. The answer can be verified thus—

$$OA^2 + OB^2 = OP^2$$
$$\therefore\ 14^2 + (11\tfrac{1}{4})^2 = OP^2$$
$$(196 +) + (127 +) = OP^2$$
$$324 = OP^2$$
$$18 = OP$$

EXAMPLES ON CHAPTER III.

1. Find the resultant graphically in the following cases, and sketch the arrangements for equilibrium—

$P = 14 \quad Q = 13 \quad \text{angle} = 35°$
$P = 5 \quad Q = 8 \quad \text{angle} = 125°$

2. In what case is $R^2 = P^2 + Q^2$ true? If a boy pulls a string to the

west with a force of 10 lbs., another to the north with a force of 10 lbs., and a third to the south-east with a force of 15 lbs., will there be equilibrium? If not, what force should the third boy have employed that there might be equilibrium?

3. Two forces inclined at an angle of 120° act on a point. One force is three times the other. Their resultant is 56. Find the forces, and show the forces and resultant in a diagram. What would be the magnitude and direction of the resultant if one force were to act towards the point, and the other from it?

4. If two boys pull at a nail by means of strings, one to the north with a force of 5 lbs., and the other to the west with a force of 7 lbs., what is the total pull on the nail, and in what direction will the nail begin to move off if it is pulled from its socket by the boys' strings?

5. Resultant = 25 : P = 20 Q = 15. Find the angle between P and Q, and show the 3 forces in a diagram. Would the particle on which they act be at rest? If not, why not?

6. State the principle of the parallelogram of forces. Forces of 7 and 11 units act on a point A, from A to P, and A to Q respectively. P A Q is an angle of 120°. Find the resultant of the two forces by construction or otherwise.

7. What do you mean by a component? What are the rectangular components of a force?

8. Find the components of a force of 16 lbs. along two lines, one at 30° to the force, and the other at 60° on the other side of it.

9. What is meant by the resolution of a force into rectangular components? A force of 18 units acts along a line, making an angle of 30° with a given line; find by construction or otherwise its components along and at right angles to that line. [*S & A.*

10. A force of 50 units acts along a line inclined at 30° to the horizon; find by construction or otherwise its horizontal and vertical components. [*S. & A.*

CHAPTER IV.

POLYGON OF FORCES.

By using the principle of the Parallelogram of Forces we can find the resultant of any number of forces acting on the same point and making angles with each other. Take the following as an illustration. Let forces P, Q, S, T of magnitudes 9 lbs., 6 lbs., 3 lbs., 12 lbs. respectively act on point O in directions indicated by arrow-heads. First choose a convenient unit of length, say $\frac{1}{6}$ inch, to represent the unit of force, 1 lb.; on O P set off by scale O A equal to $1\frac{1}{2}$ inch; on O Q set off O B equal to 1 inch; complete parallelogram O A C B; draw diagonal O C: this we know represents in magnitude and direction the resultant R_1 of P and Q. On O S set off O D

equal to $\frac{1}{2}$ inch and complete parallelogram O C E D and draw diagonal O E, which gives us the resultant R_2 of R_1 and S.

Lastly, on O T set off O F equal to 2 inches and complete parallelogram O E F G and draw diagonal O G, giving us resultant R of R_2 and T.

The resultant R is the resultant required.

Noticing the thick lines in the figure, we observe that by omitting the dotted lines and those representing R_1 and R_2 we should have obtained the same result by using the following construction. From any point draw lines parallel to the direction of and proportional to the several forces; from the end of one of these lines draw a line equal and parallel to the next; from the end of the line thus drawn another equal and parallel to the next. Continue this until we have drawn a line equal and parallel to the last of the forces. We have now two cases to consider.

1. If the construction gone through has produced an open polygon (*i.e.* an uncompleted polygon), the line needed to complete the polygon will be equal and parallel to the required resultant; in the figure this is the line O G.

2. If our construction has produced a closed polygon (*i.e.* a completed polygon), we conclude that the forces have no resultant, or, in other words, the forces are in equilibrium.

The statement of the result found is *that when any number of forces acting on a point can be represented in magnitude and direction by the sides of a polygon taken in order, they will be in equilibrium.* The converse of this is also true, viz. *that any number of forces acting on a point which are in equilibrium can be represented in magnitude and direction by the sides of a polygon taken in order.*

This is called the Polygon of Forces.

It will be seen that the word *any* in the above may mean *three*, and in this case we have the Triangle of Forces. The word *three* is substituted in the above for the word *any*, and the word *triangle* for the word *polygon.*

As this is a very important proposition, it will be well for us to state it particularly. It is: *If three forces acting on a point can be represented in magnitude and direction by the three sides of a triangle taken in order, these forces are in equilibrium.* Its converse, also true, is: *If three forces acting on a point are in equilibrium, they can be represented in magnitude and direction by the three sides of a triangle taken in order.*

Thus if the three forces P, Q, R acting on point O are in equilibrium, they can be represented in magnitude and direction by the three sides of the triangle $p\,q\,r$.

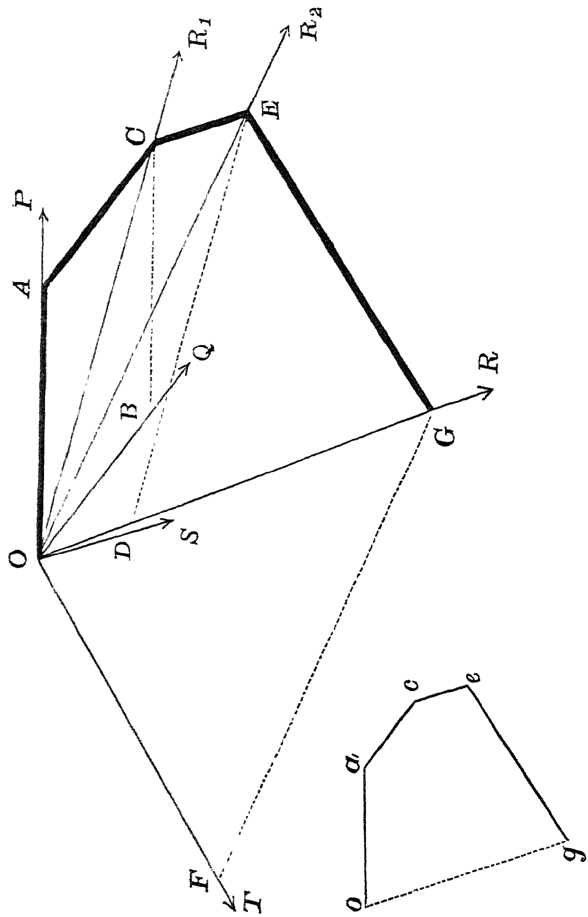

Fig. 12.

Where the side pq is parallel and proportional to the force P
" " pr " " " R
" " rq " " " Q

It will be well for us here to notice again the difference between the words *resultant* and *equilibriant*. In the figure for the polygon of forces the *resultant* of the forces P, Q, S, T is represented in magnitude and direction by the line O G; the force which, together with P, Q, S, T, keeps the point O at rest, *i.e.* which balances P, Q, S, T, or their *equilibriant*, is

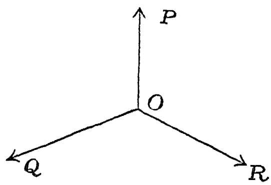

FIG. 13.

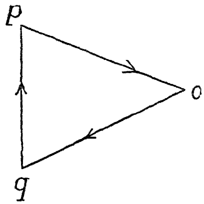

FIG. 14.

a force represented in magnitude and direction by G O; *i.e.* it is a force equal and opposite to the resultant.

In the consideration of the propositions stated, viz.: Parallelogram, Triangle, and Polygon of Forces, it must be clearly understood and remembered that if these propositions are to be used—

1. The words *taken in order* must not be overlooked.

2. The forces must act on the *same point*. *E.g.* in the figure used for the polygon of forces, the forces P, Q, S, T act on the same point O. In the figure used for the triangle of forces, the forces P, Q, R act on the same point O. A little thought will be sufficient to show that the condition (2) just mentioned can be stated thus: that the *directions* of the forces considered must pass through the same point.

EXAMPLES ON CHAPTER IV.

1. What do you understand by the triangle of forces, the parallelogram of forces, and the polygon of forces?

2. Why could not 3, 3, and 7 poundals maintain equilibrium? Examine the following sets, and see which could maintain equilibrium: 7, 8, 9; 9, 9, 18; 2, 5, 6; 3, 3, 3; 1, 0, 1; 2, 5, 9; 7, 8, 16; 5, 3, 4.

3. Arrange forces of 3, 7, and 8 lbs. to act at A, and maintain the particle A at rest. Indicate the angles between the forces.

4. If forces of 5, 7, and 10 units act on a point, show by a diagram how they must be adjusted so as to be in equilibrium. State the principle in dynamics called the triangle of forces.

5. State the rule for finding the resultant of two forces acting on a

point. Two forces of 10 units each have a resultant of 5 units; find by construction the angle between the directions of the two forces. State distinctly the result you arrive at.

6. What do you know about the arrangement of 3 forces acting on a body and maintaining equilibrium?

7. Three forces act at a point and keep it at rest; show how to draw a triangle whose sides shall represent the forces.

8. Forces of 7 and 16 units have a resultant of 21 units; find the direction of the forces by a construction drawn to scale. [*S. & A.*

9. Draw two lines, A B and A C, containing an angle of 120°, and suppose a force of 7 units to act from A to B, and one of 10 units from A to C. Find by construction the resultant of the forces and the number of degrees in the angle its direction makes with A B. [*S. & A.*

10. If forces of 4 and 12 act at 120°, what would be the effect of replacing the 4 by a force of 8 units?

CHAPTER V.

MOMENT OF A FORCE.

A MAN moves a block of stone by means of a crowbar. He has two ways of bringing more power to bear on the stone—

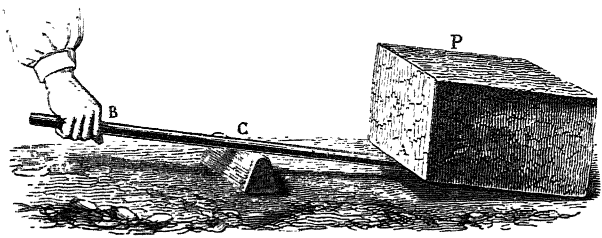

FIG. 15.

1. By increasing his own force.
2. By lengthening that arm of the crowbar which he is using.

Again, in the case of the boys' "see-saw" mentioned previously, the boy at either end has two ways of increasing his effect.

1. By pressing down on the beam with more force.
2. By increasing the distance between himself and the middle block.

These two simple examples will help us to understand one method of measuring the effect of a force.

Thus, in the figure, suppose A B a rod turning upon the pivot F, P and Q weights attached to it at points A and B respectively.

The effect of Q on A B will be to turn it about F in the direction similar to that of the hands of a watch. This effect

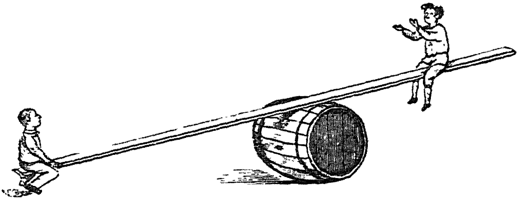

FIG. 16.

depends upon the magnitude of Q and the length of B F combined, and therefore one measure of its effect can be expressed by the product of Q and B F, *i.e.* Q × B F.

Similarly, the effect of B on A B will be to turn it about F in the opposite direction. This effect depends upon the magnitude of P and the length of A F combined, and therefore one measure of its effect can be expressed by the product of P and A F, *i.e.* P × A F.

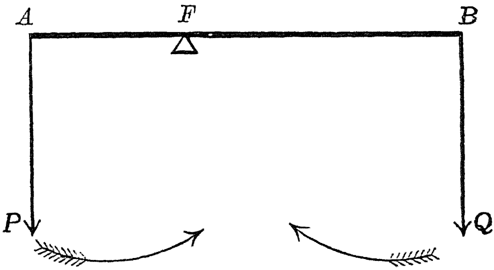

FIG. 17.

The effect of Q on A B is called the moment of Q with respect to the point F; similarly, the effect of P on A B is called the moment of P with respect to the point F.

The moment of a force acting on a body with respect to a given point is the measure of the tendency of the force to turn the body about the given point.

As the body is capable of turning about the given point in

two directions, we must have some way of distinguishing them; this is done by calling the direction similar to the direction of the hands of a watch *negative;* that in the opposite direction *positive.*

Thus the moment of W about the point C is positive.

,, ,, P ,, ,, ,, negative.

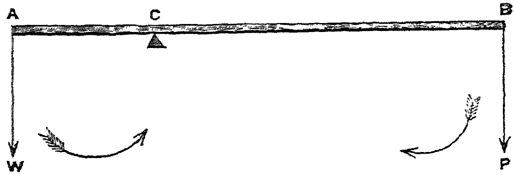

FIG. 18.

Let us examine this part of the subject a little more fully.

We meet with many instances of forces acting on bodies, where the directions of the forces do not pass through a common point; how shall we measure their effect?

This can be done by using a measure of the force called its *moment.*

Thus, let O be a given point in a body, P a force acting on the point in direction AP not passing through O. If we let fall a perpendicular O N upon A P we have found the measure of the tendency of the force P to turn the body round O. This tendency can be measured by the product P × O N from what has already been said. This product is called the moment of the force respect to the point O.

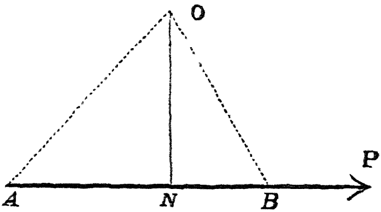

FIG. 19.

It is the product of the force multiplied by the perpendicular distance of the point from the direction of the force.

As we can represent the magnitude of the force by a line, this moment can be expressed by an area, viz. twice the area of the triangle A O B, where A B represents the force P.

It should be noticed that when the direction of the force passes through the given point, the moment of the force with respect to the given point becomes zero, as will at once be seen from the fact that the tendency of the force to turn the body about the given point is zero, or from the fact that, though

the force itself may be great, yet the perpendicular distance from the given point to the direction of the force is zero.

The "moment" may be represented by the product $P \times p$, where P is the given force, and p the perpendicular.

In the case under consideration we have $p = 0$;

$$\therefore \text{ moment of force} = P \times 0 = 0$$

Example.—A force of 5 lbs. acts on a body at a distance of 12 feet from a point O, round which the body can turn. What is its moment about O?

Here the magnitude of the force is 5 lbs., and its perpendicular distance from the point is 12 feet; its tendency to turn the body about O is measured by the product—

$$5 \times 12 = 60,$$

which is therefore the moment required.

In the figure the man exerts a force of 1 cwt.; the length of the arm is 4 feet. The moment in this case is measured by the product of the force of 112 lbs. and the perpendicular distance 4 feet;

$\therefore$ tendency to turn body about centre of capstan

$$= 112 \times 4 = 448.$$

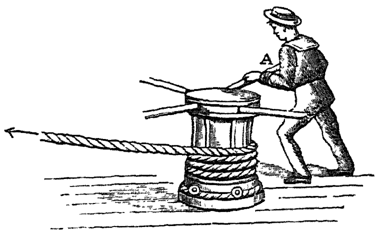

FIG. 20.

We will state here the following properties of moments, which will be readily understood from the preceding:—

1. When a number of forces act upon a body, the sum of the moments of the forces with respect to any point, their proper signs being affixed, is equal to the moment of the resultant of the forces with respect to the same point.

2. If the forces are in equilibrium, the sum of the moments, their proper signs being affixed, is zero.

Good illustrations of the use of "the moment of a force" are furnished by the mechanical power called the lever. It will be well for us to give a brief explanation.

THE LEVER.—This is a rigid rod or bar which can turn about a fixed point called the *fulcrum*. By the word *rigid* we mean that the bar or rod must not bend when loaded with weights at the two ends.

The several parts to be considered are—

1. The power. 2. The weight. 3. The arms of the lever, or the parts of the bar lying between the power or the weight and the fulcrum.

If the arms are in one straight line, the lever is said to be a *straight lever;* if not, it is said to be a *bent lever.*

Levers are divided into three classes, according to the rela-

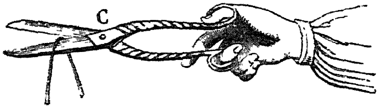

FIG. 21.

tive positions of the points of application of the power and weight with respect to that of the fulcrum.

In levers of the first kind, such as the ordinary crowbar, the

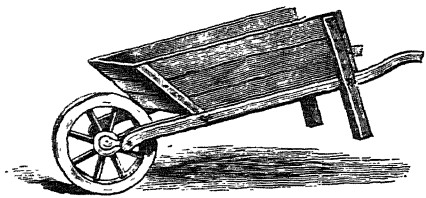

FIG. 22.

balance, the boy's "see-saw," etc., the fulcrum lies between the power and the weight.

A pair of scissors forms a double lever of the first kind, the fulcrum being the joint.

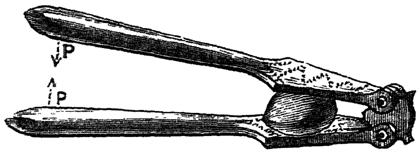

FIG. 23.

In levers of the second kind, such as the wheelbarrow, where the barrow is the weight, the wheel the fulcrum, and the power is applied at the handle, we have the power and the weight both on the same side of the fulcrum, but acting in opposite directions, the power being further from the fulcrum than the weight.

A pair of nutcrackers furnishes an example of a double lever of the second kind.

In levers of the third kind, such as the treadle of a grindstone, the power and the weight act on the same side of the

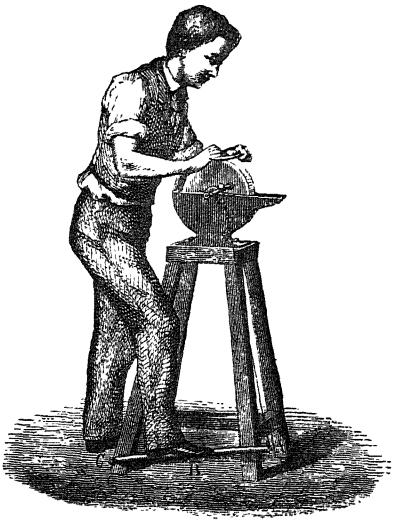

FIG. 24.

fulcrum, in opposite directions, the power being nearer the fulcrum than the weight.

The shears used for cutting grass afford an example of a double lever of the third class.

CONDITIONS OF EQUILIBRIUM OF THE LEVER.—We must examine these under two cases.

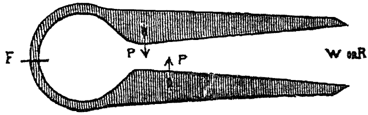

FIG. 25.

1. When the lever is straight and the forces act perpendicularly to the lever.

Here, since P and W are parallel, we at once obtain the

following, calling the force or reaction which the fulcrum exerts in opposition to the resultant, or the pressure which the resultant of P and W exerts upon the fulcrum R, and remembering that these two forces must be equal and opposite, R must be parallel to P and W, and—

in levers of the 1st kind $R = P + W$.
„ „ 2nd „ $R = W - P$.
„ „ 3rd „ $R = P - W$.

Also as the moment of P about the fulcrum must equal the moment of W about the same point, we have

$$P \times AF = W \times BF. \quad \therefore P : W :: BF : AF,$$

which proportion gives us the lengths of the arms.

Example.—A straight lever 20 inches long weighs 15 ozs. Where must the fulcrum be placed in order that the lever may be in equilibrium when a weight of 16 ozs. is hung at one end and a weight of 9 ozs. at the other?

What is the pressure on the fulcrum when the lever is in equilibrium? [*Lond. Matric., June*, 1868.

(i.) To find pressure on fulcrum—

Pressure = P + W + weight of lever.
„ = 16 ozs. + 9 ozs. + 15 ozs.
„ = 40 ozs.

(ii.) To find position of fulcrum. Here we have to remember the weight of the beam, which acts from its c.g., *i.e.* its mid-

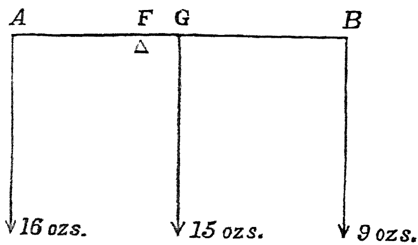

FIG. 26.

point, and is an additional force acting on one side of the fulcrum. Taking moments with respect to A, we have moment of resultant force about A = sum of moments of separate forces about A;

i.e. $40 \times x = 16 \times 0 + 15 \times 10 + 9 \times 20$
where x = distance of fulcrum from A at
$40\,x = 150 + 180 = 330. \quad \therefore x = \frac{330}{40} = \frac{33}{4} = 8\frac{1}{4}.$
$\therefore$ Fulcrum is situated $8\frac{1}{4}$ in. from A.

2. When the lever is bent and the forces act in any direction on the lever.

In this case, as the forces are not parallel, one condition, by the triangle of forces, is that the three forces P, W, and R, acting on the lever, must pass through the same point; also that the reaction R of the fulcrum must be equal and opposite to the resultant of P and W.

This resultant is at once found by the parallelogram of forces.

Examples on Chapter V.

1. Define "moment of a force with respect to a point." What units do you use, and in what units will the answer be expressed?

2. Take a square of 4 ins. side A B C D. Forces of 10 lbs. act along A B, B C, C D, and A D. Find the moments with respect to A of each force, and find the sum of the moments with respect to the centre.

3. What is the use of the + and − sign in moments? If the moments = 0 with respect to a point A, what inference do you make about the forces?

4. Define the moment of a force with respect to an assigned point. A uniform rod rests in a horizontal position on two supports 8 ft. apart, one under each end; it weighs 6 lbs. A weight of 24 lbs. is hung to it from a point distant 3 ft. from one end; find the pressure on each point of support. [*S. & A.*

5. A B C D is a square of 2 ft. side; forces of 10 and 12 lbs. act from A to D and A to B respectively; find (*a*) the algebraical sum of moments with respect to C; (*b*) what point in B C must be fixed if the forces are to balance each other about it. [*S. & A.*

6. Define the moment of a force with respect to a point. If two forces balance each other on a weightless rod capable of turning freely round a fixed point, what relation must exist between the forces? C is a point in a weightless rod (A B), round which it is capable of turning freely; A C is one-third of A B. A force of 10 units acts at right angles to A B, and is balanced by an equal force acting at B; find how the second force must act, and the magnitude of the pressure on the fulcrum. [*S. & A.*

7. When is a moment reckoned positive, and when negative? Draw an equilateral triangle A B C, 4 ft. side. A force of 8 units acts from A to B, and one of ten from C to A; (*a*) find the moment of each force with reference to the middle point of B C; (*b*) find the point with reference to which the forces have equal moments of opposite sign. [*S. & A.*

8. A B C D is a rectangular board which can turn freely in a vertical plane round a hinge at A; A B is 10 ft. long, and B C is 6 ft. The body weighs 300 lbs.; it rests with A B against a fixed point P vertically below A, and at a distance of 8 ft. from A; find (1) the moment of the weight with respect to A; (2) the reaction of P against the body.

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CHAPTER VI.

PARALLEL FORCES.

WE now come to forces acting upon a body, where the directions of the forces do not pass through a common point, but are parallel to one another.

Instances of such are found in—

1. The common balance;
2. The steelyard;
3. The wheel and axle;
4. The pulley;

and in many other forms.

All cases of such forces can be divided into two classes.

1. Where the forces act in the same direction or towards the same parts, such forces are called *like* forces.

2. Where the forces act some in one direction and the remainder in an opposite direction, or where the forces act towards opposite parts, such forces are called *unlike* forces.

I. To obtain the resultant of two parallel forces acting in the same direction.

We can best arrive at this experimentally by using the following simple apparatus.

Take a rod of wood or iron of uniform thickness and breadth. Find its middle point by suspending it by means of a looped string. From its middle point towards both ends mark on it divisions of inches, feet, etc.

Suspend it by means of the looped string from its middle division. At points at equal distances on either side of the middle division attach two equal weights, say, of 1 lb. each. Pass the string over a pulley.

It will now be found that if equilibrium is to be maintained we must attach a weight of 2 lbs. to the other end of the string.

The 2 lbs. thus attached to the string is the equilibriant of the two separate pounds attached to the rod, which of course tells us that the resultant of the two separate pounds must be a weight of 2 lbs. acting on the same point, but in the opposite direction to the equilibriant.

The simple experiment can be changed indefinitely. Thus, still using the middle division already obtained, instead of

attaching equal weights, use unequal weights, say, of 1 lb. at A, and 3 lbs. at B. We shall now need 4 lbs. at the end of the string. We also find that the points where the weights were attached in the last experiment will not be the right ones here, for the rod does not rest horizontally. Instead of these points we find we must take point B at 1 inch distance from point where loop is fixed, and point A at 3 inches distance on the other side of the loop.

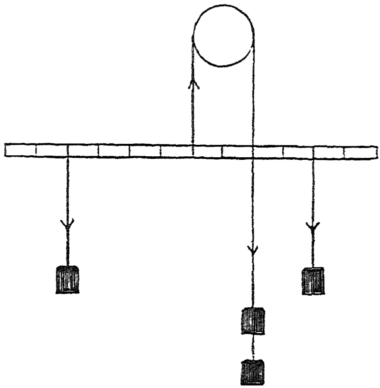


FIG. 27.

In other words, if loop be at point F, B F must be 1 inch, A F must be 3 inches, or the point F divides A B into lengths which are inversely proportional to the forces.

Let us illustrate this further.

Suppose 20 ozs. be attached at point B, and 16 ozs. at A. Then, if A F be made 1 foot, B F must be $\frac{16}{20}$ ft. That is, the length A B will be divided in the following manner by the point F—

A F : B F :: : 20 : 16.

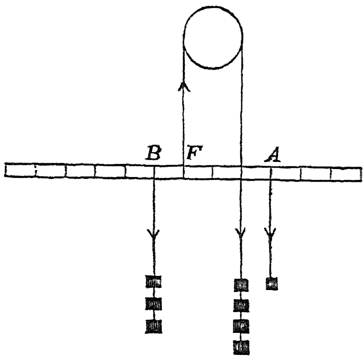


FIG. 28.

We have thus found by experiment the three following principles by which we obtain the magnitude, direction, and point of application of the resultant of two like forces.

1. Magnitude. The resultant of the two forces equals the sum of the forces, or $R = P + Q$.

2. Direction. The direction of the resultant of the two forces is parallel to that of the two forces and towards the same parts.

3. Point of application. The point of application of the resultant divides the distance between the points of application of the forces into lengths inversely proportional to the forces; *i.e.* if P act at A, and Q at B, and the resultant R at F, we have—

$$P : Q :: B F : A F.$$

The third principle follows at once from what has been said about the moment of a force. Thus in figure, if P and Q be

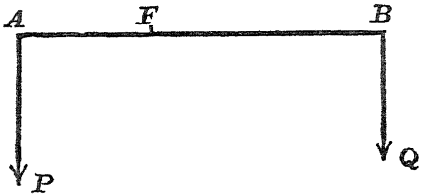


FIG. 29.

forces in equilibrium acting on the bar A B at points A and B, suppose the bar to be capable of turning about the point F.

Because there is equilibrium we have moment of P about point F = moment of Q about point F, *i.e.* $P \times A F = Q \times B F$;

$$i.e.\ P : Q :: B F :: A F.$$

Notice that in all the experiments, and of course in the principle found, the point of application of the resultant is always nearer the greater force.

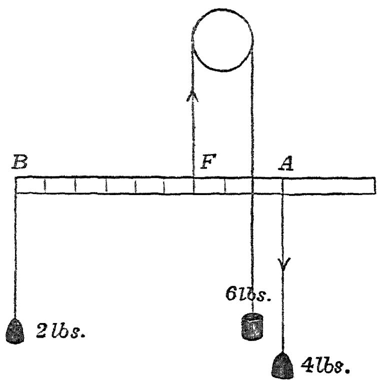


FIG. 30.

II. To obtain the resultant of two parallel forces acting in opposite directions.

The same kind of experiment may be used. Thus, to the bar used before, having found its middle division, attach at one end A, 3 inches from F, a weight, say, of 4 lbs. Attach the looped string at the middle division, pass the string over the pulley, and at the end

of the string fasten a weight, say, of 6 lbs. Here we have two forces acting on the rod towards opposite directions, one (the weight of 4 lbs.) vertically downwards, the other (a weight of 6 lbs.) vertically upwards. What is the magnitude and direction of the resultant? Where is its point of application? By experiment we shall find that to produce equilibrium we must affix a weight of 2 lbs. at a point B, 6 inches from F. This force acts vertically downwards, and is the equilibrium of the forces 4 lbs. applied at A, and 6 lbs. applied at F. Therefore our resultant must be a force equal and opposite to this equilibriant, and is a force of 2 lbs. acting at the point B in the same direction as the larger force of 6 lbs., *i.e.* vertically upwards.

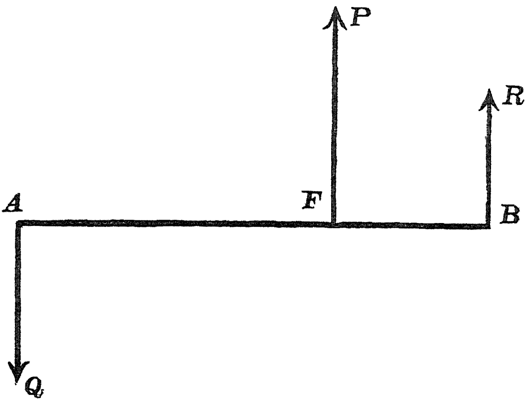


FIG. 31.

Notice here the difference between this and the other experiments. Call the weight of 4 lbs. Q, and that of 6 lbs. P.

1. The magnitude of the resultant force is equal to the difference of the two forces, *i.e.* $R = P - Q$.

2. The direction of the resultant force is the same as that of the larger force.

3. Its point of application lies outside the part of the rod between the two forces; it is nearer the larger force, and is found by the proportion—

$$Q : P :: FB : AB.$$

Here, again, we might have used the principle of moments, for if the two forces P and Q are in equilibrium, and the rod be capable of turning about point B, the moment of the force Q about the point B must equal the moment of the force P about point B;

$$\textit{i.e.}\ Q \times AB = P \times FB,$$
$$\textit{i.e.}\ Q : P :: FB : AB.$$

We will now work out an example of each kind.

Example 1. *Like Forces.*—Parallel forces of 5 lbs. and 15 lbs. act in the same direction at the ends of a rod 1 foot long. Find the magnitude and point of application of the force which will keep the rod at rest.

1. The magnitude of the resultant equals the sum of the forces. Therefore it is—

$$(5 + 15)\ \text{lbs.} = 20\ \text{lbs.}$$

The force required is therefore one of 20 lbs. acting in the direction opposite to the forces.

2. To find its point of application. Let A B represent the rod.

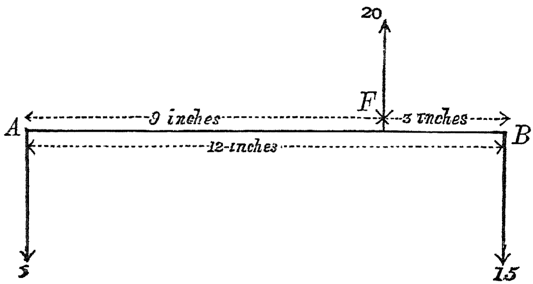


FIG. 32.

If 5 lbs. acts at A, 15 lbs. at B, and the resultant at F, we must have—

$$5 : 15 :: BF : AF;$$
$$1 : 3 :: BF : AF.$$

Let $BF = x$, then $AF = (12\ \text{inches} - x)$;

$$\therefore 3x = 12\ \text{inches} - x;$$
$$4x = 12\ \text{inches};$$
$$x = 3\ \text{inches}.$$
$$\therefore BF = 3\ \text{inches};$$
$$AF = 9\ \text{inches}.$$

We could have divided A B into 20 equal parts, and then set off 5 parts from the point B, giving F the point of application required, so that—

$$B\,F = \frac{5}{20} \text{ of } A\,B = \frac{5 \times 12}{20} \text{ inches} = 3 \text{ inches};$$

$$A\,F = \frac{15}{20} \text{ of } A\,B = \frac{15 \times 12}{20} \text{ inches} = 9 \text{ inches};$$

or, working by the principle of moments, we should have—

$$5 \times A\,F = 15\,B\,F;$$
$$A\,F = 3\,B\,F,$$

as above.

Example 2. *Unlike Forces.*—Two parallel forces of 15 lbs. and 5 lbs. act on a rod A B in opposite directions. The distance A B

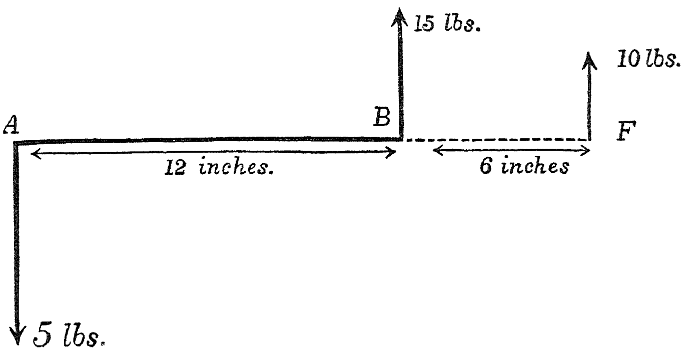


FIG. 33.

between their points of application being 1 foot, find the magnitude and point of application of their resultant.

1. Magnitude of resultant—

$$R = P - Q;$$
$$\therefore R = (15 - 5) \text{ lbs.} = 10 \text{ lbs.}$$

The resultant is a force of 10 lbs. acting in the direction of the larger force of 15 lbs.

2. Point of application of resultant.

The rule found by experiment says that the resultant acts at a point in the line produced in the side nearer the greater force.

Let F be the point required in A B produced.

Then we have—

$$Q : P :: B\,F : A\,F;$$
$$\therefore 5 : 15 :: B\,F : A\,B + B\,F;$$
$$1 : 3 :: B\,F : 12 \text{ inches} + B\,F;$$
$$3\,B\,F = 12 \text{ inches} + B\,F;$$
$$2\,B\,F = 12 \text{ inches};$$
$$B\,F = 6 \text{ inches}.$$

Therefore F is 6 inches from B.

We could have found this by actual measurement or by the principle of moments.

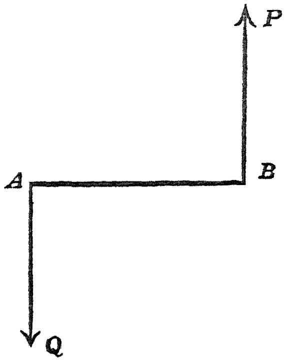


FIG. 34.

Thus by the principle of moments if x be the length of the produced part and F be the point about which we take moments, we have—

$$5\,[AB + x] = 15x;$$
$$AB + x = 3x;$$
$$12 \text{ inches} + x = 3x;$$
$$x = 6 \text{ inches},$$

as before.

One special case of parallel forces acting on a body in opposite directions should be noticed. It is when the forces under consideration are equal.

It will be seen that the effect of such a combination will be to produce rotation. If we attempt to find their resultant by the preceding we obtain—

$$R = P - Q. \quad \text{But } P = Q; \; \therefore R = 0.$$

There is, therefore, no single force by which we can replace this combination.

The only method of producing equilibrium is by applying to the body a similar combination which will produce an equal rotation in the opposite direction.

Such a pair of forces is called *a couple*, the perpendicular distance between the two forces being the *arm* of the couple.

To find the resultant of a number of parallel forces acting on a body.—This can be done by repeated applications of the two preceding propositions.

First find the magnitude and point of application of the resultant R of any pair of forces, then take that resultant and find the magnitude and point of application of the resultant of R, and as third force, and so on.

To illustrate, take the following numerical example. Find the magnitude and point of application of the weights 3 lbs., 4 lbs., and 5 lbs. suspended from a rod of 12 inches long in such a manner that 7 inches separate the weights 3 lbs., and 4 lbs., and five inches separate the weights 4 lbs. and 5 lbs.

Take weights 3 lbs. and 4 lbs.; their resultant $R = (3+4)$ lbs. $= 7$ lbs. It acts at a point D, such that—

D B = 3 inches, and A D = 4 inches.

Take R = 7 lbs. and the weight of 5 lbs.; their resultant

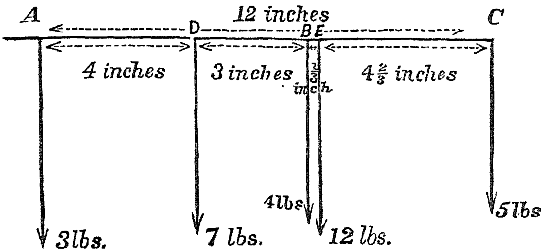


FIG. 35.

R = (7 + 5) lbs. = 12 lbs. It acts at a point E, such that—

$$D E = \tfrac{5}{12} \text{ of } D C = \tfrac{5}{12} \text{ of } (D B + B C) = \tfrac{5}{12} \text{ of } (3 + 5) \text{ inches}$$
$$= \tfrac{5}{12} \text{ of } 8 \text{ inches} = 3\tfrac{1}{3} \text{ inches};$$
$$C E = \tfrac{7}{12} \text{ of } D C = \tfrac{7}{12} \text{ of } 8 \text{ inches} = 4\tfrac{2}{3} \text{ inches}.$$

Therefore total resultant is a force of 12 lbs. acting at a point E, $4\frac{2}{3}$ inches from the weight of 5 lbs.

The point of application E of the resultant is called the *centre of the parallel forces.*

It might have been found by the principle of moments thus—

Let E be the required point. Take this as the point about which the rod can turn. For equilibrium the sum of the moments of the forces about this point must equal zero.

Let C E = x inches.

Then moment of weight 3 lbs. about E = $+3 \times AE$;

„ „ 4 lbs. „ = $+4 \times BE$;

„ „ 5 lbs. „ = $-5 \times CE$.

(Note carefully the signs of the moments.)

$$\therefore +3 \times AE + 4 \times BE - 5 + CE = 0,$$
$$i.e.\ 3(12 - x) + 4(5 - x) - 5x = 0;$$
$$36 - 3x + 20 - 4x - 5x = 0;$$
$$56 = 12x;$$
$$4\tfrac{2}{3} = x.$$

Therefore, as before, D E = $4\frac{2}{3}$ inches.

So far we have been supposing that the forces acted at right angles to the rod. Let them still be parallel, but act not at

right angles to the rod, but at some other angle. We shall still have the same resultant, and we shall find that the point of application is not affected by the direction of the forces.

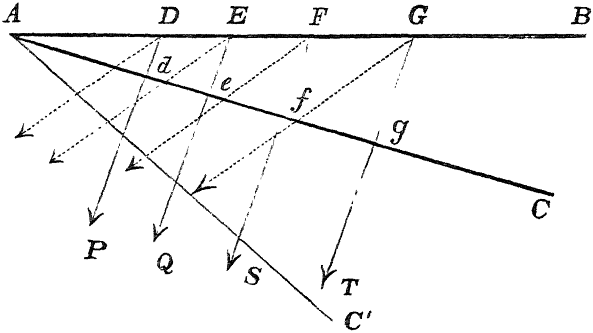


FIG. 36.

Thus let forces P, Q, S, T act on A B in direction as in figure.

Draw line A C at right angles to the direction of the forces. We are able at once to determine the resultant of the forces with its point of application. This resultant will act at right angles to A C, and therefore parallel to the given forces. Suppose the direction of the forces to be changed into direction represented by dotted lines; it will be found that the points of application of the forces being unaltered, and the forces still remaining parallel, the final resultant will always pass through the same point, *i.e.* the point of application of the resultant is independent of the direction of the forces.

EXAMPLES ON CHAPTER VI.

1. What do you mean by the centre of parallel forces? Give another name for it. What is the distinctive feature of this centre?
2. Write out formulæ, and make diagrams, for all the cases you can find of parallel forces.
3. Where is the resultant when the parallel forces act in unlike directions? and where, when they act in like directions?
4. Where is the resultant situated when two unlike parallel forces become equal? What is a couple? What is its effect on a body?
5. Show that, when three parallel forces are in equilibrium, each force is proportional to the distance between the other two.

6. Find the resultant, and its point of application, of parallel forces 5 and 7 lbs. respectively, acting in like directions at end of a bar 6 ft. long; likewise for unlike forces of 5 and 7.

7. A bar of wood 30 ins. long is capable of turning on a pivot. A boy at one end pulls a string at right angles to the bar with a force of 10 lbs., and a boy at the other end does similarly with a force of 480 poundals. How far is the pivot from the first boy, supposing that these boys do not produce rotation?

8. Suppose, in the previous arrangement, that the pivot is so placed that the second boy pulls the bar round in his direction; how would you stop this rotation (1) by changing the magnitude of forces, (2) by altering the position of the pivot?

9. Explain the principle followed in working a problem such as the following: There are two forces of 10 and 20 lbs. acting on a bar of wood in unlike parallel directions. The resultant acts at a distance of 12 ft. from the 10 lb. force; what is the length of the bar of wood?

10. Two forces of ten units each act on a body along parallel lines, and in opposite directions; why would it be impossible to balance these forces by any one force? [*S. & A.*

11. Two parallel forces of 3 and 4 units act in opposite directions; specify the force required to balance them, and show in a diagram how the forces act. [*S. & A.*

12. Take a bar of wood 2 ft. long; suppose 10 lbs. act at one end, and 12 lbs. at the other, parallel, but in opposite directions; where is their centre? What force will balance them? How must it act? Make a diagram of the three forces. [*S. & A.*

13. Define the centre of parallel forces. Forces of 5 and 7 poundals act in the same direction along parallel lines at points 2 ft. apart; where is the point of application of the resultant? If the 5 poundals be reversed, where is now the centre? Make a diagram for each case. [*S. & A.*

14. A stiff pole 12 ft. long sticks out from a vertical wall; it would break if a weight of 28 lbs. were hung at one end; how far may a boy of 8 st. venture before it breaks?

15. Take a uniform rod 10 ft. long and weighing 10 lbs.; put a support under it 2 ft. from one end, and another in the same horizontal line 3 ft. from the other end. Find the total weight carried by the supports; and find the pressure on each support separately.

16. Parallel forces of 6, 5, 4 ozs. act in like directions at points 4 ins. apart, 6 ins. and 9 ins. from one end of a line; find the centre.

CHAPTER VII.

PROPERTIES OF MATTER.

BEFORE proceeding with examples of forces it will be well to discuss certain properties of matter, as divisibility, porosity, elasticity, etc.

DIVISIBILITY.—The chalk I use will serve as an example of this property. As I write on the blackboard I leave thereon

the chalk mark which shows that the chalk has been divided into a very great number of very minute particles. I continue to use the chalk and find my fingers become covered with chalk dust, and after a time the chalk has all been used up. The stones which have been used to mend the roads are ground up into very fine dust-particles, showing that the stones can be divided into an immense number of such particles. The file on the grinder's stone is ground until it is quite smooth; as you notice the sparks flying from the stone you know that both the file and the stone are being divided into very small parts. The iron filings used in the experiments in magnetism, the sand on the sea-shore, and a very large number of instances of a like kind, all illustrate this property of matter called *divisibility*, or the property by which a body is capable of being *divided* into a large number of distinct parts.

We cannot tell by actual experiment whether there is a limit to the divisibility of matter, but the teaching of chemistry would lead us to believe that all bodies are made up of very minute and indivisible particles to which the name of *atoms* is given.

COMPRESSIBILITY.—By means of the hydraulic press, cloth, paper, hay, cotton, and other substances may be *compressed*, *i.e.* they may be made to occupy less volume than before the use of the press.

Take a stout piece of glass tubing of 1 in. bore, at one end fit a cork tightly, into the other end fit a tightly fitting

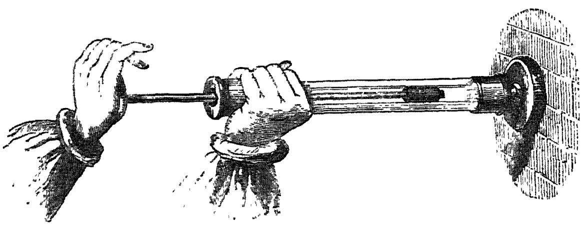


FIG. 37.

piston. By putting pressure on this piston the air inside the tube may be made to occupy one-half, one-third, etc., of its original volume.

Take a piece of watch-spring; bend it. The parts on the concave side of the spring will be compressed, whilst those on the convex side of the spring will expand.

These illustrations, together with many others of a like

kind, show that bodies may be more or less compressed, *i.e.* that they possess the property of *compressibility*.

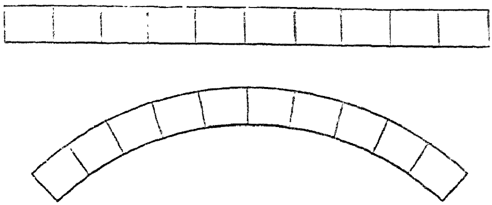


FIG. 38.

Some forms of matter can be compressed much more than others.

Gases are very compressible, the least increase of pressure producing at once a decrease of volume. Solids can be compressed to some extent, as shown by the impression of the die on the penny, the sixpence, or the half-sovereign; whilst liquids are practically incompressible: it is on this fact that the principle of the hydraulic press depends.

POROSITY.—Why are bodies compressible? This question forces us to look into the subject more closely. Take an ordinary brick and allow it to lie in a bucket full of water for a short time. We shall find that a large quantity of the water has disappeared, whilst the brick is very little larger, but weighs more. The brick is *porous*: it is said to have "sucked up" the water. The common sponge acts in the same way. Take a small quantity of water and an equal quantity of sulphuric acid. Mix the two. We should expect that the mixture would occupy double the volume of either liquid, but it is not so, for it occupies less than the expected volume.

Blotting-paper, bread, cloth, and other bodies "suck up" the water, ink, etc., surrounding them. I put my piece of chalk into a glass of water; I find the water is drawn into the chalk. Look at the skin of my hand through the magnifying-glass. I see an immense number of small openings, which are called *pores*.

All the instances given show that, though the particles of matter making up the different bodies are very near together, yet there are small spaces left between them, which spaces we call *pores*. The bodies are said to be *porous*, or to possess the property of *porosity*.

These spaces tell us where the water, ink, etc., already mentioned has gone to, and also that because the spaces of pores can be made less by pressure the bodies are compressible.

The existence of pores in metals was proved by the "Florentine" experiment.

A hollow ball of silver was filled with water and then closed up. An attempt was then made to compress the water. During the experiment the water was found to have made its way to the outer surface of the ball, showing that it had found a passage owing to the *porosity* of the silver.

ELASTICITY.—I take an ordinary indiarubber ball and throw it or allow it to fall on the floor, on which I have sprinkled evenly a small quantity of chalk-dust. It bounces back into my hand. I examine it and I find that instead of a small part of the surface of the ball being chalked, as we might expect, there is a considerable part of the ball thus marked.

How is this?

The weight of the ball bringing it into contact with the floor has compressed and flattened it, and so has brought a considerable part of its surface in contact with the chalk. But the ball, although it has been flattened, has been able to regain its original shape.

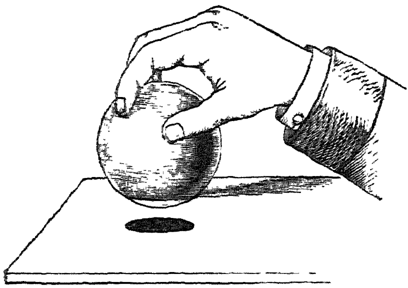


FIG. 39.

It has regained its shape, because it possesses the property of *elasticity*. As we have seen, all bodies can be compressed more or less; they also have the power, in greater or less degree, of recovering their original shapes and sizes when the compressing force is taken away.

This property we call *elasticity*.

Whilst compression is going on there is another force acting, which the body possesses because of its elasticity, called the force of *restitution*, *i.e.* the force tending to bring the body back to its original shape and size.

The ratio which the force of restitution bears to the force of compression is called the *coefficient of elasticity* of the body;

this ratio has been found to be always the same, whatever may be the magnitudes of the forces of compression and restitution, for the same substance.

If this coefficient is zero, the body is said to be *inelastic;* if the coefficient is a proper fraction, *i.e.* some quantity between zero and unity, the body is said to be *imperfectly elastic;* if the coefficient be unity, the body is said to be *perfectly elastic.*

All bodies belong to the class *imperfectly elastic.* The other terms are sometimes used, and bodies are said to be inelastic or perfectly elastic when such is not really the case.

Other properties of matter which we must mention briefly are tenacity, ductility, and malleability.

Tenacity is the resistance which a body possesses against any force which tends to pull its particles apart.

Ductility is the property which bodies possess which are capable of being drawn out into wire.

Malleability is the property which bodies possess which are capable of being rolled or hammered out into a sheet without breaking. Perhaps the best illustration of this property is afforded by gold-leaf, which is used for gilding. This can be beaten out so thin as to be almost transparent.

CHAPTER VIII.

CENTRE OF GRAVITY.

In a previous chapter we have defined the term *weight;* it will be well for us here to look at the term again.

Sir Isaac Newton established a law which deals with every particle of matter in the universe, viz. that every particle of matter possesses the power of *attracting* every other particle of matter to itself: this law is called the law of *universal gravitation.*

This law of universal gravitation expresses the fact that all bodies are mutually acting upon each other, no matter what their state may be, whether of rest or of uniform motion.

The exact amount of this force of attraction between two particles is stated thus. *The attraction between two material particles is inversely proportional to the squares of their distances apart and directly proportional to the product of their masses.*

To explain this, take a case of two spheres, A and B.

(i.) With regard to their distances apart, disregarding their masses.

1.	Let A be	1 foot distant from B,	we may call the force of attraction a						
2.	,,	2 feet	,,	,,	the force of attraction will be				$\frac{a}{2^2}$
3.	,,	3 feet	,,	,,	,,	,,	,,		$\frac{a}{3^2}$
4.	,,	4 feet	,,	,,	,,	,,	,,		$\frac{a}{4^2}$

etc.

i.e. When these spheres are separated by			1 foot	attraction is	a
,,	,,	,,	2	,,	$\frac{a}{4}$
,,	,,	,,	3	,,	$\frac{a}{9}$
,,	,,	,,	4	,,	$\frac{a}{16}$

(ii.) With regard to their masses, neglecting distances.

1.	Let A be	1 lb.	and B	1 lb.,	call force of attraction b		
2.	,,	2 lbs.	,,	3 lbs.,	the force of attraction		$= b \times 2 \times 3$
3.	,,	4 ,,	,,	5 ,,	,,	,,	$= b \times 4 \times 5$
4.	,,	6 ,,	,,	7 ,,	,,	,,	$= b \times 6 \times 7$

The two parts of the law can be brought together under one expression thus—

Choose a certain length as unit of distance, and a certain mass as unit of mass; also let the unit of attraction be that which would exist between two particles of unit mass separated by unit distance; then the force of attraction between two bodies of mass m and m' respectively separated by distance d is $\frac{m \times m'}{d^2} = \frac{m\,m'}{d^2}$.

Example.—Compare solar gravity with terrestrial gravity.

We have to find here the weights of two equal masses, one on the surface of the earth, the other on the surface of the sun.

Assuming that the masses of the earth and sun may be supposed to be collected at their centres, and that the mass of the sun = 355,000 times the mass of the earth, also remembering that the distance in each case is the radius of each sphere, we have

$$\text{solar gravity} : \text{terrestrial gravity} :: \frac{355000 \times E}{(440000)^2} : \frac{E}{(4000)^2}$$

the radius of the earth being 4000 miles,

,, ,, sun ,, 440000 ,,

i.e. solar gravity : terrestrial gravity :: $\frac{355,000}{(110,000)^2} : \frac{1}{(1000)^2}$

" : " :: 3550 : 121

" : " :: 29·3 : 1

i.e. 1 lb. on the earth's surface would weigh 29·3 lbs. on the sun's surface.

The universal law of gravitation affects all bodies on the surface of the earth. When speaking of the force of gravitation in connection with the earth and bodies at or near its surface, we use the term *gravity;* for example, we say that, because of the force of gravity, all heavy bodies tend to fall to the ground. A stone thrown vertically upwards descends after a time vertically downwards; the rain-drop, the snow-flake, the hail-stone, all fall to the ground because they are acted upon by the force of gravity. This is true, no matter how small the body acted upon may be.

Taking, therefore, the particles into which a body may be divided, we see that every particle of which the body is composed is acted upon by this force of gravity.

The direction of this force is in all cases towards the centre of the earth; yet, as the centre of the earth is, comparatively speaking, at such a great distance, we may consider the forces acting upon the separate particles of a body as parallel. If we find the centre of these parallel forces we shall have found a point in the body called the *centre of gravity of the body.*

Take as an illustration a common brick; this can be ground into very minute particles.

We know that each particle has a certain weight, and if the weights of the whole of the particles be added together we shall obtain the weight of the brick. Suppose also that we could suspend each particle by a fine thread; the threads used will all take one direction, that of the force of gravity, and our supposed experiment will show us that the forces acting on the particles are all parallel in direction.

Examine the same particles as they compose the brick; the same forces are acting, and these forces are parallel.

From what we have learnt about parallel forces we know that in the case of the parallel forces under consideration there is one point at which the resultant of the parallel forces may be supposed to act.

This point is called *the centre of gravity of the brick.*

I hold a slate in my hand. I can support it either at the four corners or I can find one point about which I can make

it spin. At this point I have applied a force which is equal and opposite to the resultant of the parallel forces acting on the separate particles of the slate. This point of support must therefore be directly underneath the centre of gravity of the slate.

The determination of the centre of gravity of a body is a very useful problem in mechanics.

We can find the centre of gravity of many bodies either—

1. Experimentally; or
2. By geometrical construction.

1. *Experimentally.* Let us take as our first example the slate referred to.

Suspend the slate by a point in one of the sides by means of a string. From this point draw a chalk line on the slate vertically downwards, which of course will be the continuation

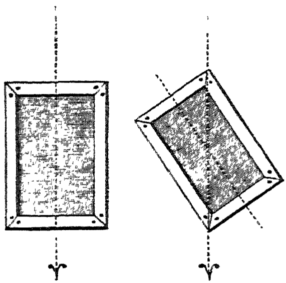


FIG. 40.

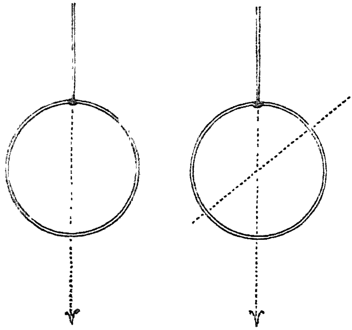


FIG. 41.

of the line of the string. This vertical line can easily be drawn on the slate by means of a plumb-line. The resultant of the many forces acting on the particles of the slate is a force acting vertically downwards, and as the body is in equilibrium under the action of two forces, viz. the tension of the string and this resultant, these two forces must be equal, and must act in the same straight line, but in opposite directions. Therefore we know that the point of application of this resultant, *i.e.* the centre of gravity of the body, must be somewhere in this chalk line. Suspend the slate at one of the four corners and proceed as before. We now find a second line in which the c. g. must be. Therefore the c. g. required must be at the intersection of the two lines.

As a second example, take a boy's hoop. Deal with it as

in the case of the slate, but represent the chalk lines by strings. We shall get two intersecting lines as before, the c. g. of the hoop being at their intersection. Notice the difference in the two cases.

In the slate the c. g. is in the material of the body; in the hoop it is not.

Other examples where the centre of gravity is not in the substance of the body are found in the wedding ring, the hollow sphere, the skeleton cube, the surface of the pyramid, etc.

2. Let us now look at the other method. Here we will confine our attention to *homogeneous* bodies, *i.e.* bodies of uniform density throughout. This is the same as saying that if equal volumes be taken from any part of the body, they will be found to be of the same weight.

As the weights of the separate parts of the body represent the amounts of the forces of gravity acting upon them, it will be seen that the problems we have to solve can be included under the head of parallel forces.

Also, in considering the c. g.'s of lines, surfaces, and solid bodies, we can take the lengths of the lines, the areas of the surfaces, and the volumes of the bodies as being proportional to the weights of the parts under consideration.

We can therefore find by geometry the centres of gravity of all regular figures, including straight and curved lines, triangles, rectangles, the circle, the ellipse, the sphere, the cube, cone, pyramid, etc.

We will now work out a few simple examples.

1. Two equal particles placed at a distance apart. Considering them as one body, we have two equal and parallel forces acting on the body at the two given points. By parallel forces these two forces can be replaced by one single force equal to their sum, its point of application being midway between the two points. This is therefore the c. g. required.

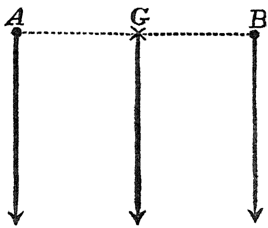


FIG. 42.

FIG. 43.

2 An odd number of particles equal in weight, placed so

as to be in a straight line, and at equal intervals. Let A, B, C, D, E, F, H, be the particles.

Taking	A and H	together,	their c. g.	is at	D
„	B and F	„	„	„	D
„	C and E	„	„	„	D

Therefore the c. g. of the whole is at D, *i.e.* at the mid-point of the line formed by joining A H.

3. A straight line. From the last example we at once gather that the c. g. of a straight line is at its middle point.

4. Four particles placed at the corners of a square. Take two of them at opposite corners, say A and C. By example (1) their c. g. is at G, the mid-point of A C.

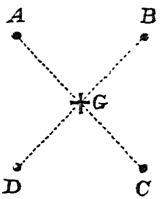


FIG. 44.

Similarly, c. g. of B and D is at G.

∴ c. g. of the four particles is at the intersection of the diagonals of the square.

5. Combining examples (3) and (4), we find that the c. g. of the perimeter of a square or rectangle is at the intersection of the diagonals.

6. An even number of particles of equal weight placed so as to form a circle. Here, by taking two particles which are the extremities of one diameter, we find the c. g. of these two is at the centre of the circle, and so with the others, taking them in pairs; from which we at once find that the c. g. of the whole is at the centre of the circle.

This helps us to the c. g. of a circle, which is at its geometrical centre.

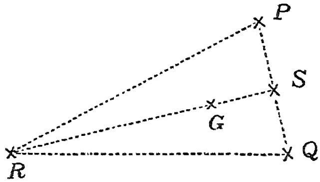


FIG. 45.

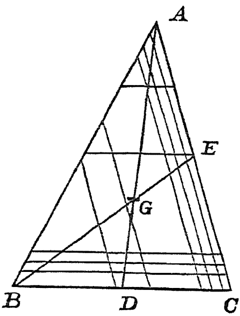


FIG. 46.

7. As in the last case, we can find the centre of gravity of a hollow sphere which is at the geometrical centre.

8. Three particles placed at the angles of a triangle. Here we have two parallel and equal forces acting at points P and Q. Their resultant must equal their sum, and must act at point S, which bisects P Q. We have therefore to find the point of application of the resultant of two parallel forces acting at R and S, the one at S being twice the magnitude of the one at R. By parallel forces this will be the point G, where G divides R S in the proportion

$$RG : GS :: 2 : 1$$
$$i.e.\ GS = \tfrac{1}{3} RS$$

9. The centre of gravity of the area of a triangle. Let A B C be the given triangle. Bisect B C in D; join A D. The c. g. of the triangle must be in A D, for this line bisects every line drawn parallel to B C, and must therefore pass through the c. g. of each line making up the triangle.

Bisect A C in E; join B E.

The c. g. of the triangle must be in B E, for similar reason that c. g. is in A D.

The c. g. must therefore be at G, the intersection of the two lines.

By easy geometry, we find that G E = $\frac{1}{3}$ B E.

The same result would have been obtained if we had suspended the triangle by angle A, and then by angle B, and thus have found experimentally the required c. g.

The principle of moments will help us to find the c. g. in many cases.

What has been said about moments will include the following. If a body be divided into a number of separate parts, the moment of the whole about any point is equal to the sum of the moments of the separate parts about the same point.

A few examples will help us to understand the use of this principle.

1. Particles of weights 12, 10, 8 ozs. respectively are placed in a straight line, so that the weights of 12 and 10 ozs. are

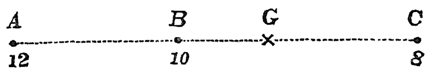


FIG. 47.

separated by a distance of $2\frac{1}{2}$ feet; those of 10 and 8 ozs. are separated by a distance of $5\frac{5}{8}$ feet. Find the distance of the c. g. of the whole from the largest weight.

We have the following equation—

Moment of whole about A = sum of moments of parts about A,

$$i.e.\ (12 + 10 + 8) \times x = 12 \times 0 + 10 \times AB + 8 \times AC$$

where x = distance of c. g. from A

$$\therefore 30\,x = 0 + 10 \times 2\tfrac{1}{2} + 8 \times 8\tfrac{1}{8}$$
$$\therefore 30\,x = 0 + 25 + 65$$
$$\therefore 30\,x = 90$$
$$\therefore \quad x = 3$$

$\therefore$ A G = 3 feet, where G is c. g. required.

2. Two circles touch each other externally. Their radii are 3 feet and 4 feet respectively; their weights are proportional to their areas. Find the position of their common centre of gravity.

Note.—The area of a circle $= \pi r^2$

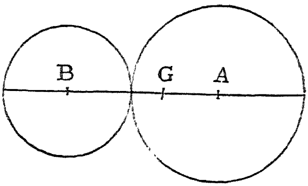


FIG. 46.

Where $\pi = \frac{22}{7}$, r = radius of circle.

Equation to use is—

Moment of whole about A = sum of moments of parts about A,

$$i.e.\ \{\pi \times 3^2 + \pi \times 4^2\} \times x = \{\pi \times 4^2\} \times 0 + \{\pi \times 3^2\} \times AB$$

where x is the distance from A of c. g.

remembering that the c. g. of a circle is its geometrical centre, and that therefore the weight of each circle may be supposed to act at its centre.

$$\therefore \pi \times 25\,x = 0 + \pi \times 9\,AB$$
$$25\,x = 9 \times 7, \text{ for } AB = 7$$
$$x = \frac{63}{25} = 2\frac{13}{25}$$

$\therefore$ A G $= 2\frac{13}{25}$ feet, where G is c. g. required.

3. Find the common centre of gravity of four particles of equal weight placed at the four angles of a triangular pyramid. By a previous example the common c. g. of A B C is at E, where E F $= \frac{1}{3}$ B F. Join D E.

Consider the forces acting at the points D and E; we have one force acting at D and one three times as great acting at E; therefore G, the common c. g. required, divides D E in the following proportion—

$$DG : GE :: 3 : 1$$
$$i.e. \ 3\,GE = DG$$
$$i.e. \quad GE = \frac{1}{3} DG$$
$$\therefore \quad GE = \frac{1}{4} DE$$

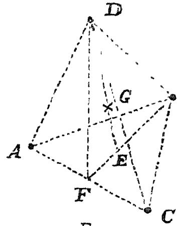


FIG. 49.

This problem will help us to remember the position of the centre of gravity of a solid pyramid; it is in the line joining the apex with the centre of the base, at a distance from the base equal to one-fourth of the line joining the apex with the centre of the base.

This applies to a pyramid with any rectangular base. A solid cone has its c. g. in the same relative position, *i.e.* in the line joining the apex to the centre of the base, one-fourth of the line from the base.

The surface of the cone has its c. g. in the same line, but one-third of the line from the base, as can be deduced from the position of the c. g. of the triangle.

It will be seen that we can find the c. g. of an irregular rectangular figure by dividing the figure into triangles, finding the c. g. of each triangle, and then treating the problem as a case of parallel forces.

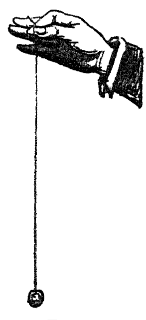


FIG. 50.

Before leaving this section it will be well to define the terms *vertical* and *horizontal*. The direction of the force of gravity at any point of the earth's surface is called the *vertical direction* at that point.

It is the perpendicular to still water. The bricklayer when using the plumb-line is really finding the vertical line at the point where he is at work.

As will be seen, the vertical direction points to the centre of the earth, so that a vertical line, say at Sheffield, will not be parallel to a vertical line at London, since the two would meet if produced.

The horizontal direction is the one at right angles to the

vertical. It is the surface that a *limited* quantity of water assumes when at rest.

STATES OF EQUILIBRIUM.—It will be seen that the centre of gravity of a body is a very important point in the body, because it is that point at which the whole weight of the body may be supposed to act.

Consider the case of a body resting on a plane surface; this body is acted on by two forces.

1. Its own weight, vertically downwards, which acts from the c. g. of the body.

2. The reaction of the plane, which acts at right angles to the plane, and at some point within the base of the body, or within the line drawn round the points of support of the body. Our knowledge of forces acting thus gives us the two following conditions of equilibrium, viz. :—

1. The two forces must be equal.

2. They must pass through the same point.

Let us look at this a little closer.

A body is said to be in equilibrium when the several forces acting upon it balance one another, and so produce rest.

There are three kinds of equilibrium:

(i.) Stable.
(ii.) Unstable.
(iii.) Neutral.

If the body be in such a position that on being slightly displaced it returns to its original position, the equilibrium is said to be *stable*.

If the body be in such a position that on being slightly

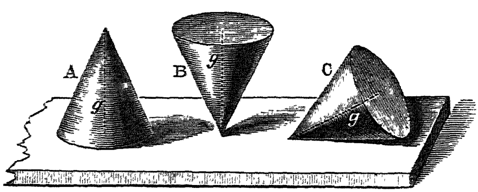


FIG. 51.

displaced it tends to move farther away from its original position, the equilibrium is said to be *unstable*.

If the body be in such a position that on being slightly

displaced it still remains in equilibrium, it is said to be *neutral.*

Perhaps the best illustration of these three stated is obtained from the cone.

The cone on its base is an instance of a body in *stable* equilibrium.

The cone on its apex is an instance of a body in *unstable* equilibrium.

The cone on its side is an instance of a body in *neutral* equilibrium.

Other instances of stable equilibrium are the cube on one side, the cylinder on one end.

Instances of unstable equilibrium are seen in the cube on one edge, the egg on either end, the loaded cart placed as in the figure.

Bodies in neutral equilibrium are the sphere, the cylinder on one side.

In the case of stable equilibrium the vertical line let fall from the centre of gravity of the body falls inside the base after a slight displacement. Therefore the weight of the body tends to bring the body back to its first position. The builder places the ends of his scaffold-poles in barrels, in order to give them wider bases, so that this vertical line may fall inside the base in all cases.

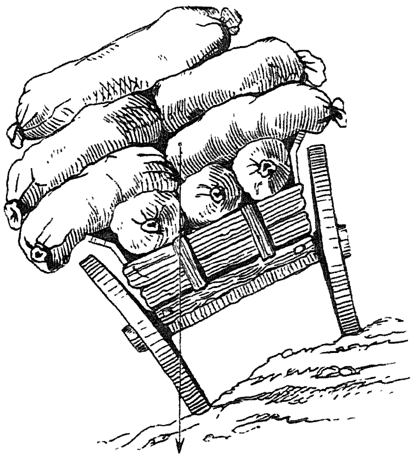


FIG. 52.

The c. g. in stable equilibrium is low down.

A sphere of wood loaded with lead will be in stable equilibrium when the lead is in its lowest position.

In unstable equilibrium the vertical line dropped from the c. g. of the body does not fall inside the base after a slight displacement. Therefore the weight of the body tends to carry

the body further from its original position. The c. g. here is high up. The loaded sphere will be in unstable equilibrium when the lead is in its highest position.

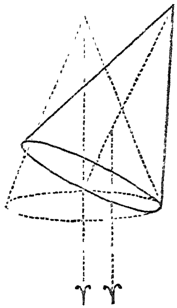


FIG. 53.

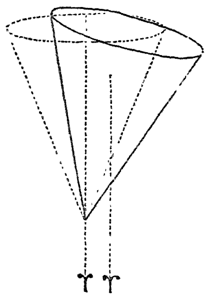


FIG. 54.

Let us tabulate the different facts concerning the three states of equilibrium.

States of Equilibrium of a Body.			
Stable ...	Vertical line dropped from c. g. falls inside base after slight displacement of body.	c. g. of body low down.	Wide base.
Unstable	Vertical line dropped from c. g. does not fall inside base after slight displacement of body.	c. g. of body high up.	Narrow base.
Neutral ...	Vertical line dropped from c. g. always falls inside base.	c. g. of body always in same relative position.	Base always the same size.

Examples on Chapter VIII.

1. What do you understand by "the centre of gravity"? Mention an experimental way of showing that the c. g. of a circular board is at its centre. [*S. & A.*

2. If a rod of iron balances on a certain point, what inference do you draw regarding this point?

3. A lever having a weight of 8 lbs. attached to one end, and 17 to the other, balances about a certain fulcrum. Where must the centre of gravity be if we are justified in leaving the weight of lever out of the question?

4. How do you proceed to find the c. g. of a uniform rod of wood?

Explain the principles involved. What do you mean by the c. g. of "a line"?

5. Where is the c. g. in each of the following cases? Sketch the figures, and indicate the c. g. by the intersection of lines in each case: (1) Equilateral triangle, (2) scalene triangle cut out of sheet-iron, (3) a piece of wood half an inch thick, rectangular in shape, (4) a cone, (5) a square prism, (6) a triangular pyramid, (7) any pyramid, (8) a row of tin triangles, scalene, isosceles, or right-angled, standing between the same parallels.

6. Describe an experimental method of finding the c. g. of a slate, a circular piece of tin, a walking-stick. Explain each step carefully. [*S. & A.*

7. Where is the c. g. of a billiard ball? Take a smooth, perfect, solid ball; suppose that every time it is put on the table it rests on one particular spot on its surface: what inference do you draw from this?

8. When a body is placed on a plane, and is acted on only by gravity and the reaction of the plane, what condition will determine whether it will topple over or not?

9. Mention any one property of the centre of gravity of a body. A 1 oz. weight is placed at the corner of a tin triangle weighing 1 oz. Where is the c. g.? Make a diagram to show this distinctly.

10. What are the three kinds of equilibrium? Give an example of each. Drive a tack through the c. g. of a tin triangle into a wall so that the triangle can spin freely round. What kind of equilibrium have we here? Now put the tack through it a short distance nearer the base. What are the two positions of equilibrium now?

11. If you had a circular tube, bent into the form of a ring, held in an upright position, through which a marble runs, what positions and what kinds of equilibrium can you arrange for?

12. What is the object of having ballast on board a vessel? Why should a cart containing a ton of iron be safer, other things being equal, than a ton of hay in the same cart?

13. A body is in shape a sphere, but loaded in such a way that its centre of gravity is not at its geometrical centre: when it is placed on a horizontal table what are its positions of stable and unstable equilibrium? [*S. & A.*

14. A cube is placed on a horizontal table: in what positions is it in stable, and in what positions in unstable, equilibrium? [*S. & A.*

15. If you know the c. g. of each part of a compound body, how will you find the c. g. of the whole? Take for example a T square. The cross-piece is 2 ins. wide, and weighs 3 ozs.; the shaft is 36 ins. long, and weighs 3½ ozs. Find the distance of the c. g. from the far end of the shaft.

16. Two rods of uniform density are put together so that one stands on the middle point of the other at right angles to it; the former weighs 3 lbs., and the latter 2 lbs. Find the c. g. of the whole. [*S. & A.*

17. How would you find, experimentally, the c. g. of an irregular, flat, heavy plate? Make a diagram to explain your process. If the plate were held in various positions, would this affect the position of the c. g.? What property of the c. g. justifies your answer? [*S. & A.*

18. Find the c. g. of the following compound body: bar of wood 5 ft. long, weight 3 lbs., balancing on a point 2 ft. from one end; at this end a 4 lb. weight is fastened; at the other, a 5 lb. weight; and at the centre, 1 lb.

19. Where is the c. g. of three equal weights placed at the corners of a

triangle? Why is this? If one weight be removed and placed with one of the remaining two, where is now the c. g., supposing each weight is 4 ozs. and the triangle also weighs 4 ozs.?

20. A weight of 4 ozs. is placed at each of three corners of a square: where must a weight of 8 ozs. be placed so that the c. g. of whole is at intersection of diagonals?

21. Write out a rule for finding the c. g. of two weights placed a certain distance apart; use it to find the c. g. of 4 and 5 ozs. placed 3 ft. apart.

22. Take a square of 8 ins. side, and weighing 4 lbs.; divide this into four equal squares; indicate the c. g. of each square in a diagram. Now remove one of the squares; find the c. g. of the remainder.

23. A circular piece of cardboard weighing 5 grammes, radius 10 centimetres, has a hole of radius 5 centimetres punched out of it, the centre of the larger circle being on the circumference of smaller: the weights being proportional to square of the radius, where is now the c. g.?

24. A tapering rod 3 ft. long balances about a point one-third of its length from one end. If a weight double its own weight be hung from the thinner end, about which point will it now balance?

25. Where is the c. g. of a square piece of cardboard situated? If the cardboard weighs 1 oz., and a half-ounce weight is placed at one corner, where will the c. g. of the whole be? [*S. & A.*

CHAPTER IX.

WORK.

A LABOURER carries eight bricks, each weighing 5 lbs., up a ladder to a scaffold, the height of which is 15 feet. Besides raising his own weight, he has overcome the resistance of the force of gravity upon the weight of 40 lbs. through the height of 15 feet. He is said to have done a certain amount of *work.* This amount is measured by the product of the resistance overcome into the distance through which the resistance is overcome. The unit used is called the *foot-pound.*

As will be seen, the term "resistance overcome" is not confined to the resistance of the force of gravity experienced when weights are lifted: it includes resistances of every kind, such as the resistance offered to the nail as it is driven into the beam; the resistance to compression by the hydraulic press as different materials are subjected to its pressure; the resistance of friction as it prevents the rough brick from sliding down the rough incline, etc.

In measuring the amount of work done we must, therefore, be careful to notice the direction in which the resistance is acting, and the distance through which the resistance is over-

come must be in the direction exactly opposite to that of the resisting force.

In the case of raising a given weight the direction to be measured will be vertically upwards.

If a weight of 20 lbs. be raised 10 feet up a smooth incline whose height is 6 feet, we have to remember that the resistance acts vertically downwards, and that it is overcome through a distance of 6 feet, not 10 feet.

Bearing in mind this explanation as to the way of measuring the distance, the foot-pound may be defined as *the amount of work done in overcoming the resistance equal to* 1 *lb. through a distance equal to* 1 *foot.* Where W = weight moved, h = distance through which W is moved, $W \times h$ = total amount of work done. If W be expressed in pounds and h be expressed in feet, then $W h$ = number of foot-pounds of work done. The labourer mentioned will perform

$$40 \times 15 \text{ foot-pounds} = 600 \text{ foot-pounds.}$$

In raising the weight of 20 lbs. through a length of 10 feet along the incline,

$$20 \times 6 \text{ foot-pounds of work is done} = 120 \text{ foot-pounds.}$$

The person who walks up the staircase on to the next floor of the building does work in changing his position in opposition to the force of gravity which resists that change. He has lifted his own weight through a certain height.

The steam hammer does work as it compresses the hot metal into a smaller space.

The heat of the fire does work as it changes the water into steam, and thus alters the shape and size of the water.

The illustrations given will help us to understand what is meant when in mechanics we use the term *work.*

WORK *is the act of producing a change, either as regards size, shape, or position of a body in opposition to a force which resists that change.*

Example 1.—A man winding up his clock has to raise

(1) Two weights of 3 lbs. each through 4 feet;
(2) One weight of 2 lbs. through 4 feet;

what amount of work has he to do?

To raise the two weights through the given height he does

$$2 \times 3 \times 4 \text{ foot-pounds of work.}$$

To raise the one weight through the given height he does

$$1 \times 2 \times 4 \text{ foot-pounds of work.}$$

Therefore altogether he does

$$(2 \times 3 \times 4 + 1 \times 2 \times 4) \text{ foot-pounds}$$
$$= 32 \text{ foot-pounds.}$$

Example 2.—A bricklayer's labourer can do 337,500 units of work in a day of 8 hours long. The man himself weighs 12 stone; his hod weighs 12 lbs., and he can carry 18 bricks, each weighing 5 lbs. He has to carry his load each journey to a height of 25 feet. How many journeys up the ladder will he make in one day?

Taking the equation—

$W\,h$ = work done in one journey,
n = number of journeys, we have
$W\,h\,n$ = 337,500 foot-pounds.

Now W = his own weight + weight of hod + weight of bricks;
$\therefore$ $W = 12 \times 14$ lbs. + 12 lbs. + 18×5 lbs.;
$\therefore$ $W = 270$ lbs.;
$h = 25$ feet;
$\therefore$ $W\,h = 270 \times 25$ foot-pounds;
$\therefore$ we have

$$270 \times 25 \times n = 337{,}500$$
$$\therefore n = \frac{337{,}500}{270 \times 25} = \frac{1250}{25} = 50.$$

Ans. 50 journeys.

So far we have spoken only of the whole work done, without mentioning the rate at which the work is done.

The rate of doing work is the whole amount of work done in a given time divided by the given time. This rate is generally measured by using the term *horse-power*.

An engine is said to be able to do so many horse-power.

An engine of one horse-power is able to do 33,000 foot-pounds of work per minute, *i.e.* in one minute it can overcome a resistance of 33,000 pounds through a distance of one foot, or a resistance of 1000 lbs. through 33 feet, etc.

Example 1.—What is the horse-power of the engine which draws a train with the uniform rate of 45 miles an hour against a resistance of 900 lbs.? [*S. & A.*, 1887.

Here the engine does—

$$900 \times 45 \times 1760 \times 3 \text{ foot-pounds of work in one hour,}$$

i.e. it does—

$$\frac{900 \times 45 \times 1760 \times 3}{60} \text{ foot-pounds per minute.}$$

Therefore its rate of doing work is

$$\frac{9\cancel{00} \times 45 \times \overset{16}{\cancel{1760}} \times \cancel{3}}{60 \times \underset{\cancel{11}}{\cancel{33,000}}} \text{ horse-powers}$$

$$= \frac{9 \times 45 \times 16}{60} \text{ h.p.} = \frac{9 \times \overset{3}{\cancel{45}} \times 4}{\cancel{15}} \text{ h.p.} = 108 \text{ h.p.}$$

Ans. 108 horse-powers.

Example 2.—What is the horse-power of a waterfall of 18 feet, when the stream above the fall passes through a section of 6 square feet at the rate of $2\frac{1}{2}$ miles an hour? [*S. & A.*, 1880.

Here $h = 18$ feet.

$$2\tfrac{1}{2} \text{ miles an hour} = \frac{5}{\cancel{2}} \times \frac{\overset{44}{\cancel{1760}} \times \cancel{3}}{\underset{\cancel{2}}{\cancel{60}}} \text{ feet per minute}$$

$$= 220 \text{ feet per minute,}$$

i.e. a volume of water equal to 6×220 cubic feet falls through a height of 18 feet in one minute.

A cubic foot of water weighs 1000 ozs.

$$\therefore \text{ we have } W = \frac{6 \times 220 \times 1000}{16} \text{ lbs.}$$

$$h = 18 \text{ feet}$$

$$\therefore \text{ number of horse-powers} = \frac{6 \times \overset{\overset{5}{\cancel{20}}}{\cancel{220}} \times \cancel{1000} \times \overset{6}{\cancel{18}}}{\underset{4}{\cancel{16}} \times \underset{\cancel{3}}{\cancel{33000}}}$$

$$= \frac{\overset{9}{\cancel{36}} \times 5}{\cancel{4}} = 45.$$

Ans. 45 horse-powers.

Examples occasionally arise which are somewhat puzzling to the beginner; a little thought, however, will clear up the difficulty.

Illustration.—A well is to be dug, say, 20 feet deep and 4 feet in diameter; a cubic foot of earth weighs, say, 12 lbs. Find the amount of work to be done.

The student says to himself, "Shall I use the whole depth of 20 feet in my calculation, and consider all the earth to have been raised from the bottom of the shaft? No. If I do this my answer will be too large. Shall I calculate as though all

the earth were at the top of the well? No. This would give me a result too small. Shall I take half the depth?"

To help in obtaining a correct result let us take an easier example.

Two weights of 10 lbs. and 12 lbs. are raised through 6 feet and 4 feet respectively. Find the amount of work done.

Here our answer is—

$$(10 \times 6 + 12 \times 4) \text{ foot-pounds} = 108 \text{ foot-pounds.}$$

We should have arrived at the same result if we had found the height through which the centre of gravity of the two weights had been raised and then multiplying this height by the sum of the weights. Thus—

1. To find G H, the height through which G is raised.

This can be best done by taking the moments of the weights about line A B = sum of moments of each weight about A B.

$\therefore$ $(10 + 12) \times x = 10 \times AP + 12 \times BK$, where x = height required;

$AP = 6$ feet, $BK = 4$ feet;

$\therefore$ $22x = 10 \times 6 + 12 \times 4$;

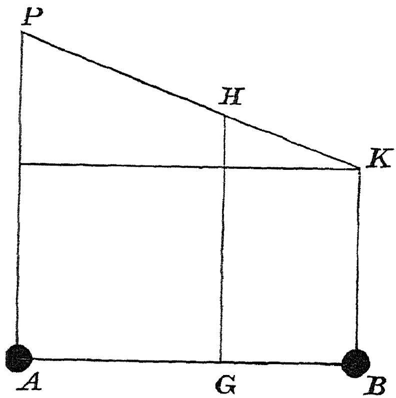


FIG. 55.

$$22x = 108;$$
$$x = \tfrac{108}{22} \text{ feet} = 4\tfrac{10}{11} \text{ feet.}$$

2. Work done $= W \times h$

$$= (22 \times 4\tfrac{10}{11}) \text{ foot-pounds} = 108 \text{ foot-pounds.}$$

We shall find this method to be correct, no matter how many weights we have to deal with. As each stratum of earth may be considered as a weight acting from its centre of gravity, we may solve all problems such as the one used as an illustration by the following rule—

When two or more weights are lifted through different heights, the amount of work done is measured by the product of

the sum of the weights multiplied by the height through which their centre of gravity has been raised.

In the case of the question of the well, its c. g. will be half-way down; therefore the amount of work done is found by the product

$W \times h$, where W = weight of earth lifted;

$$h = \tfrac{1}{2} \text{ depth of well} = 10 \text{ feet}$$
$$W = 20 \times \pi \times 2^2 \times 12 \text{ lbs.}$$
$$\text{for } \pi r^2 = \text{area of a circle}$$
$$W = 20 \times \tfrac{22}{7} \times 4 \times 12 \text{ lbs.}$$
$$\therefore W h = 20 \times \tfrac{22}{7} \times 4 \times 12 \times 10 \text{ foot-pounds}$$

Example.—Weights of 10 lbs. and 8 lbs. are connected by a very fine thread, which rests on a rough fixed pulley, so that they hang suspended. The heavier weight is found to be just not heavy enough to fall and draw up the lighter weight. If, now, we suppose the weights made to move uniformly, so that one goes up and the other runs down through 12 feet, how many foot-pounds of work are done against friction during the motion? [*S. & A.*, 1883.

The heavier weight descends, whilst the lighter ascends; where is the c. g. of the two weights at the end of the motion to be considered?

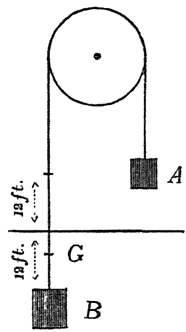


FIG. 56.

We have $8 \times AG = 10 \times BG$

$$8 \times AG = 10(24 - AG)$$
$$\therefore AG = \tfrac{240}{18} = \tfrac{40}{3} = 13\tfrac{1}{3}$$

$\therefore$ the c. g. of the two bodies has descended $= 1\tfrac{1}{3}$ foot.

$\therefore$ amount of work done $= W \times h = 18 \times 1\tfrac{1}{3}$ foot-pounds $= 24$ foot-pounds.

Example.—If a man can work at the rate of 210,000 foot-pounds an hour, how long will it take him to raise a weight of 10 tons through 150 feet, supposing him to be provided with a suitable machine? [*S. & A.*, 1891.

Work to be done is 10 tons through 150 feet
$= 10 \times 2240$ pounds through 150 feet
$= 10 \times 2240 \times 150$ foot-pounds

But he works at the rate of 210,000 foot-pounds an hour
$\therefore$ number of hours required to do work

$$= \frac{1\cancel{0} \times 224\cancel{0} \times 15\cancel{0}}{210{,}\cancel{000}} = \frac{224 \times 15}{210} = 16$$

Ans. 16 hours.

In the description of "work" given we have used the unit "foot-pound." Other units are sometimes used, *e.g.* the *kilogrammetre, which may be defined as the amount of work done in overcoming the resistance equal to one kilogram through a distance equal to one metre.*

It is an easy exercise to change the "foot-pound" into the "kilogrammetre," remembering that a kilogram = 1000 grams, and that a pound = 453·6 grams; a metre = 39·37 inches.

Foot-poundal.—If we remember the definition of a poundal, we can readily define the "foot-poundal." Taking the definition of a "poundal" as that given in the Chapter on Force, we have that *the foot-poundal is the amount of work done in overcoming, through a distance of one foot, the resistance of that amount of force which, acting for one second on one pound of matter, produces a velocity of one foot per second.*

It was found that a poundal = $\frac{1}{32}$ lb. (roughly)

$$\therefore \text{ a foot-poundal} = \frac{\text{foot-pound}}{32} \quad \text{(roughly)}$$

Examples on the use of the foot-poundal will be given later on.

Examples on Chapter IX

1. What is the mechanical definition of work? What two units must you employ in specifying a unit of work? What is the unit of work? What additional unit is introduced when we speak of the "rate of working"?

2. What is the ordinary British unit of work? Distinguish between foot-pound and foot-poundal. What is another name for this last?

3. Write out definitions of foot-pound, foot-poundal, horse-power.

4. When is a force said to do work? What is the effective work done by a force? How is the work done by a force measured? If force is estimated in pounds, and distance in feet, what is the unit employed? If grammes and centimetres, what unit then?

5. How is the working power of an agent measured? When is an agent said to work with one-horse power?

6. If a man lift a weight of 4 lbs. through a vertical distance of 9 ft., what work has to be done? Express your answer in ordinary British units, and in British absolute units. What work, if 6 lbs. through 6 ft., or 12 lbs. through 3 ft., or ½ lb. through 72 ft.?

7. What work is done in the following case: 5 lbs. lifted vertically through 4 ft., then horizontally along a smooth surface for 10 ft., then vertically through 5 ft., then, finally, along a line at 30° to horizon for 20 ft.?

8. A man weighing 10 stone carries a weight of 56 lbs. up a vertical

ladder for 12 ft.; how much work does he do in nine such upward journeys?

9. A man weighing 140 lbs. puts a load of 100 lbs. on his back, and carries it up a ladder to the height of 50 ft.; how many foot-pounds of work does he do altogether, and what part of his work is done usefully?

10. A weight of 10 lbs. is placed on a rough horizontal plane, and caused to move through 50 ft. by a pull of 4 lbs.; how many units of work have been done?

11. If a man does 1,056,000 foot-pounds of work in 8 hours, what is his rate of working?

12. If a man perform 10 journeys per hour up a ladder 50 ft. long, inclined 60° to the horizon, with a weight of $60\sqrt{3}$ lbs. on his back, what is his average rate of working?

13. Find the horse-power of an engine which draws a train against a resistance of 1000 lbs. at a uniform speed of 45 miles per hour.

14. A man draws a bucket of water weighing, say, 70 lbs. from a well 20 ft. deep; he works at the rate of $\frac{1}{100}$ of a horse-power; how many times does he draw the bucket up in 1 hour?

15. A man exerts a pressure of 30 lbs. on the arm of a capstan at a distance of 10·5 ft. from the centre of rotation; he works at the rate of 198,000 foot-pounds per hour; how many turns does the capstan make in 1 hour? [*S. & A.*

16. A pumping-engine is partly worked by a weight of 2 tons, which at each stroke of the pump falls through 4 ft. The pump makes 10 strokes a minute; how many gallons of water (1 gall. = 10 lbs.) are lifted per minute by the weight from a depth of 200 ft.? [*S. & A.*

17. What is the horse-power of a waterfall of 20 ft., if 200 galls. (1 gall. = 10 lbs.) of water fall over per minute?

18. In what time will an engine of 5 horse-power raise 5 cub. yds. of stone (1 cub. yd. = 1 ton, say) from a depth of 40 yds.?

19. Compare the power of two agents, one of which lifts 56 lbs. through 40 ft. in 2 minutes, and the other 2 cwt. through 15 yds. in 9 minutes.

20. A horse drawing a cart along a level road at the rate of 2 miles per hour performs 29,216 foot-pounds of work in 3 minutes; what pull in pounds does the horse exert in drawing the cart?

21. A body weighing 8 cwt. is drawn along 100 ft. up an incline rising 2 in 5; the resistance of friction being neglected, find the work done.

22. A hole is punched through an iron plate half an inch thick, and the pressure exerted is 36 tons; find the number of foot-pounds of work done, supposing the resistance to be uniform.

23. The surface of water in a well is at a depth of 20 ft., and when 500 galls. have been pumped out, the surface is lowered 26 ft.; find the number of units of work done in the operation (a gallon of water weighs 10 lbs.). [*S. & A.*

24. A shaft 560 ft. deep and 5 ft. in diameter is full of water; how many foot-pounds of work are required to empty it, and how long would it take an engine of $3\frac{1}{2}$ horse-powers to do the work ($\pi = 3\frac{1}{7}$)? [*S. & A.*

25. A weight of 500 lbs., by falling through 36 ft., lifts, by means of a machine, a weight of 60 lbs. to a height of 200 ft.; how many units of work have been expended in friction, and what proportion does the expenditure bear to the whole amount of work done? [*S. & A.*

26. Define a unit of work and a foot-pound. A B is a rod 20 ft. long that can turn freely round the end A. At B a force of 35 lbs. is applied at right angles to A B. The rod is allowed to turn six times; how many foot-pounds of work are done by the force? [*S. & A.*

27. A rod 7·5 ft. long can move freely in a plane round one end; it is acted on at right angles to its length by a force of 35 lbs.; find the number of foot-pounds of work done by the force in one turn. How many turns must it make a minute if the force works with one horse-power ($\pi = 3\frac{1}{7}$)? [*S. & A.*

CHAPTER X.

UNIFORMLY ACCELERATED MOTION.

So far we have been discussing forces which produce rest, or which are in equilibrium. We have now to consider the effect of forces which are not in equilibrium, or which produce motion.

In the section on MOTION we have spoken of bodies moving with—

1. A uniform velocity.
2. A variable velocity.

We now wish to discuss these terms with their meanings, and the reasons why bodies move with one or the other.

1. A uniform velocity is the velocity with which a body moves, which has been set in motion by a force which simply starts the body, and then ceases to act on the body.

This we learn from the first law of motion, which tells us that a body once set in motion will continue to move in a straight line at a uniform rate unless acted on by some external force.

2. A variable velocity will be the effect of a force acting on a body which not only sets the body in motion, but continues to act on the body, or which may give place to some other force which will act continuously on the body. To illustrate these different kinds of velocity—

(1) The ball started by a blow from the bat would move in a straight line with a *uniform* velocity if it were not acted on by the external force of gravity, the resistance of the atmosphere, the force of the wind, etc. Here the ball is set in motion by the force of the blow, which then ceases to act on the ball.

(2) The stone thrown vertically upwards has its ascending velocity constantly decreased because the force of gravity is all the time acting on the stone, tending to bring it to rest. The stone comes to rest. It then begins to descend; its descending velocity constantly increases, because now the force of gravity is acting on it in a direction tending to increase its velocity. This is therefore a case of variable velocity, due to the force of gravity constantly acting.

The railway train is brought gradually to a standstill by the continuous action of the break.

The two cases given are cases of variable velocity where the velocity increases or decreases at a uniform rate. There are instances where the increase or decrease is not uniform, but we will not discuss these at present.

We see, then, the reason for the difference between the two velocities.

In the case of uniform velocity the body moves without any interference from external forces. In the case of variable velocity the body moves under the action of external forces. As will at once be seen, all instances of motion are those where the velocity is variable.

If we wish to determine fully the motion of a body moving with a variable velocity, we must know the rate of increase or decrease in velocity.

If we have an increase or decrease, due to a force which is constantly acting, this increase or decrease will be uniform, because during equal small intervals of time equal increases or decreases of velocity will be given.

The increase or decrease of velocity when such is uniform is termed its *acceleration*, which will be *positive* when the velocity *increases*, and *negative* when it *decreases*.

A negative acceleration is sometimes called a *retardation*.

Illustration.—A body moves during five seconds with the velocities 8, 10, 12, 14, 16 feet per second respectively. Here the acceleration is positive, and is equal to 2 feet per second.

A ball rolls along smooth ice during 8 seconds with the velocities 48, 45, 42, 39, 36, 33, 30, 27 feet per second respectively. Here the acceleration is negative, and is equal to 3 feet per second.

The symbol f is used to denote an acceleration, and is generally expressed in feet per second, so that an acceleration of 5 feet per second means an increase in the velocity of a body of 5 feet in each second.

A body starts to move with a velocity V subject to a constantly acting force which imparts to the body an acceleration of f feet per second.

At the beginning	of the	1st	second	its velocity		$= V$
" end	"	"	"	"		$= V + f$
" "	"	2nd	"	"		$= V + 2f$
" "	"	3rd	"	"		$= V + 3f$
" "	"	4th	"	"		$= V + 4f$

etc.; and at the end of the tth second its velocity $= V + ft$

If $V = 0$, *i.e.* if the body starts from rest, we have the velocity v at the end of the

$$t\text{th second} = ft$$
$$i.e.\ v = ft$$

Examples.—A body starts from rest, and its velocity is increased 8 feet per second; find its velocity at the end of 8 seconds.

$$\text{Here } V = 0;\ f = 8;\ t = 8$$
$$\therefore\ v = 0 + 8 \times 8$$
$$\therefore\ v = 64$$

Therefore velocity required is 64 feet per second.

A body has a velocity of 64 feet per second at the end of 2 seconds, and a velocity of 112 feet per second at the end of 4 seconds; what is its acceleration?

Here we do not know whether the body started from rest or not, but we have the two equations—

$$64 = V + 2f$$
$$112 = V + 4f$$
$$\therefore\ 48 = 2f$$
$$\therefore\ f = 24$$

Acceleration required $= 24$ feet per second

There are several questions which have to be discussed with regard to bodies moving with a variable velocity.

To find an expression for the *space* passed through by a body moving from rest with an acceleration of f feet per second.

Consider the space passed through during a small interval of time. If we calculate on the supposition that the body moves through the whole of this interval with the velocity it has at the *beginning* of the interval, we shall obtain a result too small. Whereas, if we calculate on the supposition that the body moves through the whole interval with the velocity it has at the *end* of the interval, we shall obtain a result too great.

We should obtain a correct result if we were to add together these two results and take their mean.

The following table will help us to results which will be useful. Here we will use $f = 32$, this being the figure used in the motion of falling bodies. We should point out that the reasoning of the table would be more satisfactory if, instead of taking a second as the small interval, we were to use a much smaller interval of time.

Thus working out results for two or three cases we have—

(1) Velocity at beginning of 1st second (starting from rest) $= 0$.

∴ Space passed through during 1st second on first supposition from equation $s = vt$, we have—

$$s_1 = 0 \times 1 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

velocity at end of 1st second $= 32$, because $f = 32$.

∴ Space passed through during 1st second on second supposition from equation $s = vt$, we have—

$$s_1 = 32 \times 1 = 32 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

But (1) is too small, and (2) is too great

Taking the mean we have—

$$s = \frac{0 + 32}{2} = 16$$

(2) Velocity at beginning of 2nd second $= 32$.

∴ Space passed through during 2nd second on first supposition from equation $s = vt$, we have—

$$s_2 = 32 \times 1 = 32 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Velocity at end of 2nd second $= 64$.

∴ Space passed through during 2nd second on second supposition from equation $s = vt$, we have—

$$s_2 = 64 \times 1 = 64 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

But (1) is too small, and (2) is too great.

Taking the mean we have—

$$s = \frac{32 + 64}{2} = 48$$

and so on.

Generally velocity at beginning of tth second $= f(t-1)$;

∴ Space passed through during tth second on first supposition from equation $s = vt$, we have—

$$s^t = f(t-1) \times 1 \ldots\ldots\ldots\ldots\ldots (1)$$

Velocity at end of tth second $= f \times t$

$\therefore$ Space passed through during tth second on second supposition from equation $s = vt$, we have—

$$s_t = ft \times 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

But (1) is too small, and (2) is too great.

$\therefore$ taking mean we have—

$$s_t = \frac{f(t-1) + ft}{2}$$

$$s_t = \tfrac{1}{2}f(2t-1)$$

To find total space passed through from beginning of motion up to end of any given second.

1. To end of 1st second—

$$= 16 = 16 \times 1^2 = \tfrac{1}{2}f \times 1^2$$

2. To end of 2nd second—

$$= 16 + 48 = 16 \times (1+3) = 16 \times 4 = \tfrac{1}{2}f \times 2^2$$

3. To end of 3rd second—

$$= 16 + 48 + 80 = 16 \times (1+3+5) = 16 \times 9 = \tfrac{1}{2}f \times 3^2$$

(t) To end of tth second—

$$= 16 \times \{1 + 3 + 5 + 7 + \ldots (2t-1)\} = 16 \times t^2 = \tfrac{1}{2}f \times t^2$$

Equation (3) could have been obtained from equation (2) thus—

Space passed through during t seconds $= \tfrac{1}{2}ft^2$

" " " $(t-1)$ seconds $= \tfrac{1}{2}f(t-1)^2$

$\therefore$ " " " tth second

$$= \tfrac{1}{2}ft^2 - \tfrac{1}{2}f(t-1)^2$$
$$= \tfrac{1}{2}f\{t^2 - t^2 + 2t - 1\}$$
$$= \tfrac{1}{2}f\{2t-1\}$$

If we square equation (1) and multiply equation (2) by $2f$ we get—

$$v^2 = f^2t^2$$
$$2fs = f^2t^2$$
$$v^2 = 2fs$$

giving us (4)

Another method of illustrating these equations is by using geometrical figures; it will assist the memory somewhat if these are given. The simplest case is that for uniform velocity, viz. $s = vt$. As the right-hand side of this equation is the product of two quantities, vt, we can represent their product s, by a rectangle, of which the sides are vt.

		1st second.	2nd second.	3rd second.	4th second.	5th second.	tth (*i.e.* any) second.
1.	Velocity at beginning of ...	0	32	64	96	128	$32 \times (t - 1)$ $f \times (t - 1)$
2.	Velocity at end of	32	64	96	128	160	$32 \times t$ $f \times t$
3.	Space passed through during ... (if velocity were uniform, as in 1)	0 $s = vt$ $v = 0$	32 $s = vt$ $v = 32$	64 $s = vt$ $v = 64$	96 $s = vt$ $v = 96$	128 $s = vt$ $v = 160$	$32 \times (t - 1)$ $f \times (t - 1)$
4.	Space passed through during .. (if velocity were uniform, as in 2)	32 $s = vt$ $v = 32$	64 $s = vt$ $v = 64$	96 $s = vt$ $v = 96$	128 $s = vt$ $v = 128$	160 $s = vt$ $v = 160$	$32 \times t$ $f \times t$
5.	Space passed through during ... (taking mean of results of 3 and 4)	$\frac{0 + 32}{2}$ 16	$\frac{32 + 64}{2}$ 48	$\frac{64 + 96}{2}$ 80	$\frac{96 + 128}{2}$ 112	$\frac{128 + 160}{2}$ 144	$\frac{32 \times (t - 1) + 32 \times t}{2}$ $\frac{f \times (t - 1) + ft}{2}$ $\frac{1}{2}f(2t - 1)$
6.	Total space passed through from beginning of motion up to end of	16 16×1^2 $\frac{1}{2}f \times 1^2$	$16 + 48$ 16×2^2 $\frac{1}{2}f \times 2^2$	$16 + 48 + 80$ 16×3^2 $\frac{1}{2}f \times 3^2$	256 16×4^2 $\frac{1}{2}f \times 4^2$	400 16×5^2 $\frac{1}{2}f \times 5^2$	$16 \times t^2$ $\frac{1}{2}f \times t^2$

Thus we have found the equations
$$\begin{cases} (1)\ v = ft; \\ (2)\ s = \frac{1}{2}ft^2; \\ (3)\ \text{space during } t\text{th second} = \frac{1}{2}f(2t - 1). \end{cases}$$

Thus, draw line A B; from A mark off A C equal to v units in length. From A draw A D perpendicular to A C, and mark off A E equal to t units in length. Complete the rectangle. This rectangle will represent the product of vt, *i.e.* the space passed over in time t, when the body moves with a uniform velocity v.

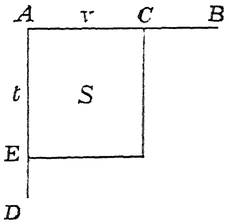


Fig. 57.

For the equation $s = \frac{1}{2}ft^2$. As before, draw line A D; on this mark A E equal to t units of length.

At equal distances in A E mark points 1, 2, 3, . . . to represent the end of different intervals of time. At each point thus marked set off lines perpendicular to A E, representing the velocities the body has at the respective points.

Thus, because the body starts from rest at point A, representing the beginning of the first interval, the velocity of the body is 0; therefore the length of this perpendicular is 0. At point 1, the beginning of the second interval, the velocity of the body is f since its acceleration is f; therefore the length of the perpendicular is f units, and so on. At point t the velocity is ft, and the length of the perpendicular is ft. If now we join the ends of the perpendiculars we get a triangle whose height is t, and base ft. This triangle represents the space passed through by the body during time t, moving with an acceleration f.

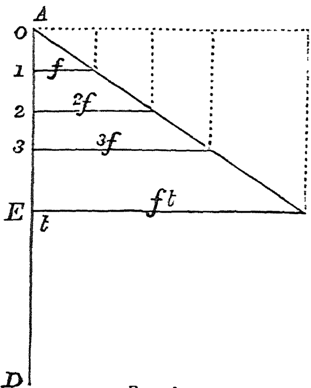


Fig. 58.

The area of the triangle is $\frac{1}{2}ft \times t = \frac{1}{2}ft^2$

$$\therefore \text{ we get } s = \frac{1}{2}ft^2$$

So far we have supposed the bodies to start from rest.

Let them now be supposed to start with an *initial* velocity V, *i.e.* a velocity V at the beginning of the motion. As this velocity V is a uniform velocity, it will not suffer any change during the motion, and we get the velocity of the body made

up of two parts, one part due to the velocity V, the other due to the acceleration f

$$\therefore v = V \pm ft \text{ .... } (1)$$

according as f is positive or negative.

The space passed through is due partly to the velocity V, giving Vt, and partly to the acceleration f, giving $\frac{1}{2}ft^2$

$$\therefore s = Vt \pm \tfrac{1}{2}ft^2 \text{ .... } (2)$$

according as f is positive or negative.

Multiply (1) by t and subtract twice (2) we get—

$$vt = Vt \pm ft^2$$
$$2s = 2Vt \pm ft^2$$
$$\therefore vt - 2s = -Vt$$
$$t(v + V) = 2s$$
$$t = \frac{2s}{v + V}$$

Substitute the value of t in (1), we get—

$$v = V \pm f\frac{2s}{v + V}$$
$$\therefore v^2 + vV = vV + V^2 \pm 2fs$$
$$\therefore v^2 = V^2 \pm 2fs \text{ .... } (3)$$

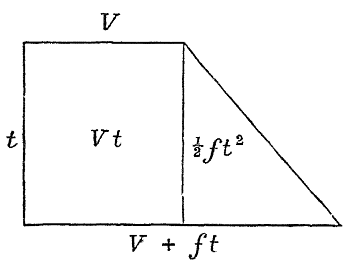


FIG. 59.

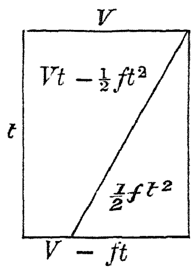


FIG. 60.

Equation (2) can be illustrated geometrically—

1. The rectangular space due to V
2. The triangular space due to f.

The one figure shows the space passed through when f is positive, the other the space when f is negative.

We will now tabulate the results obtained.

1. Bodies moving with a uniform velocity—

$$(1)\ s = vt$$

2. Bodies moving from rest with a variable velocity caused by an acceleration f.

(1) $v = ft$
(2) $s = \frac{1}{2}ft^2$
(3) space passed through during any given second $= \frac{1}{2}f(2t - 1)$
(4) $v^2 = 2fs$

3. Bodies moving with an initial velocity V, with a variable velocity caused by an acceleration f—

(1) $v = V \pm ft$
(2) $s = Vt \pm \frac{1}{2}ft^2$
(3) $v^2 = V^2 \pm 2fs$

Before working out examples on the equations found, we will give a little help on the question of change of units, as this often gives trouble to the student.

He must carefully distinguish in every problem between the terms "velocity" and "acceleration." He must remember that the word *velocity* signifies the rate at which a body moves, or is the measure of the amount of space passed over in a given time, and is found by dividing the whole space passed through by the time occupied.

The *acceleration* is the rate of increase or decrease in the velocity, *i.e.* it is the measure of the amount of increase or decrease in the velocity in a given time, so that questions on acceleration speak not only of the increase or decrease, but also of the time occupied in producing the increase or decrease. The difference between the two terms will be best seen if a few examples of each are worked out.

Thus for change of units with regard to *acceleration*, to change ft.-sec. units to yd.-min. units, we have as follows :—

In each second there is added a velocity of (say) 10 ft. per sec.

That is in each second „ „ „ $\frac{10}{3}$ yd. per sec.

„ „ second „ „ $\frac{10}{3} \times 60$ yds. per min.

„ „ minute „ „ $\frac{10}{3} \times 60 \times 60$ yds. per min.

Changing this to mile-hour units, we have—

In each minute there is added a velocity of

$$\frac{10}{3} \times \frac{60 \times 60}{1760} \text{ mile per min.}$$

$$\text{,, minute ,, } \frac{10}{3} \times \frac{60 \times 60 \times 60}{1760} \text{ mile per hour}$$

$$\text{,, hour ,, } \frac{10}{3} \times \frac{60 \times 60 \times 60 \times 60}{1760} \text{ mile per hour}$$

$$= \frac{10}{3 \times 1760} \times 60^2 \times 60^2 \text{ miles per hour}$$

i e. an acceleration of 10 ft. per sec. is equivalent to

$$\text{,, ,, } \frac{10}{3} \times 60^2 \text{ yds. per min.}$$

$$\text{,, ,, } \frac{10}{3} \times \frac{60^2 \times 60^2}{1760} \text{ miles per hour}$$

Example 1.—A velocity of 5 miles per hour is equal to

$$5 \times 1760 \times 3 \text{ feet per hour}$$

$$= \frac{5 \times 1760 \times 3}{60} \text{ feet per minute}$$

$$= \frac{5 \times 17\cancel{60} \times 3}{60 \times 6\cancel{0}} \text{ feet per second}$$

$$= \frac{88}{12} \text{ feet per second}$$

$$= 7\tfrac{1}{3} \text{ feet per second}$$

An acceleration of 5 miles per hour in each hour signifies that during each hour the body receives an increase of velocity of

5 miles per hour;

that during each minute the body receives an increase of velocity of

$\frac{5}{60}$ mile per hour;

that during each minute the body receives an increase of velocity of

$$\frac{5}{60 \times 60} \text{ mile per minute}$$

that during each second the body receives an increase of velocity of

$$\frac{5}{60 \times 60 \times 60} \text{ mile per minute}$$

that during each second the body receives an increase of velocity of

$$\frac{5}{60 \times 60 \times 60 \times 60} \text{ mile per second}$$

i.e. that during each second the body receives an increase of

$$\text{velocity of } \frac{5 \times \overset{\overset{22}{88}}{\cancel{1760}} \times \cancel{3}}{\underset{\underset{3}{\cancel{12}}}{\cancel{60}} \times 60 \times 60 \times \underset{\cancel{2}}{\cancel{60}}} \text{ ft. per second}$$

$$= \frac{22}{3 \times 3600} \text{ ft. per second}$$

$$= \tfrac{11}{5400} \text{ ft. per second}$$

Example 2.—A train is moving at the rate of 54 miles an hour. At what rate is it moving in feet per second? [*S. & A.*, 1885.

This is a question of velocity.

Therefore we have—

$$54 \text{ miles an hour}$$

$$= 54 \times 1760 \times 3 \text{ feet an hour}$$

$$= \frac{\overset{\overset{9}{\cancel{27}}}{\cancel{54}} \times \overset{44}{\cancel{1760}} \times \cancel{3}}{\underset{\underset{5}{\cancel{15}}}{\cancel{60}} \times \underset{\cancel{2}}{\cancel{60}}} \text{ ft. a second}$$

$$= 79\tfrac{1}{5} \text{ ft. per second}$$

If this question had read, "The train is moving subject to an acceleration of 54 miles an hour," we should have dealt with it thus—

Acceleration in 1 hour of 54 miles an hour

$=$ " 1 minute of $\frac{54}{60}$ mile an hour

$=$ " 1 minute of $\dfrac{54}{60 \times 60}$ mile a minute

$=$ " 1 second of $\dfrac{54}{60 \times 60 \times 60}$ mile a minute

$=$ " 1 second of $\dfrac{54}{60 \times 60 \times 60 \times 60}$ mile a second

$$= \frac{\overset{\overset{9}{\cancel{27}}}{\cancel{54}} \times \overset{\overset{11}{\cancel{44}}}{\cancel{1760}} \times \cancel{3}}{60 \times \underset{15}{\cancel{60}} \times \underset{\underset{5}{\cancel{15}}}{\cancel{60}} \times \underset{\cancel{2}}{\cancel{60}}} \text{ ft. per second}$$

$$= \tfrac{11}{500} \text{ ft. per second}$$

We will now work out a number of examples on the equations of motion.

In solving any problem involving these questions, it would

be well if the student were to consider carefully such questions as—

1. Am I dealing with a body moving from rest? If so, the question I have to use must not contain V.

2. What facts about the moving body do I know? Is it an instance of uniform motion? Is it one of accelerated motion? If so, is the acceleration positive or negative?

3. What have I to find?

Example 1.—A body whose velocity is known to be uniformly accelerated is moving at the rate of 100 yards a minute, and 10 seconds afterwards at the rate of 160 yards a minute; what is the acceleration of its velocity, and what distance does it describe in those 10 seconds? [*S. & A.*, 1880.

This is a question of velocity and acceleration. As we do not know whether the body starts from rest, we must use equation—

$$v = V + ft.$$

Also velocity v must be expressed in feet per second

$\therefore$ (1) a velocity of 100 yards a minute $= \dfrac{100 \times 3}{60}$ feet per second

$= 5$ feet per second

(2) a velocity of 160 yards a minute $= \dfrac{160 \times 3}{60}$ feet per second

$= 8$ feet per second

Therefore we now have the two equations—

$$5 = V + ft$$
$$8 = V + f(t + 10)$$

Therefore by subtraction—

$$3 = 10f$$
$$f = \tfrac{3}{10}$$

Therefore acceleration is $\frac{3}{10}$ ft. per second.

Assuming the body to start from rest, space described in the 10 seconds is obtained from

$$s = \tfrac{1}{2}ft^2$$
$$s = \frac{3}{20} \times 10^2 = \frac{3 \times 100}{20} = 15$$

Ans. 15 feet.

Example 2.—A body moving with a uniformly accelerated motion passes through 10 feet in the first two seconds after starting from rest; how far will it be from the starting-point at the end of the third second? [*Lond. Matric.*, Jan., 1868.

Here $s = \frac{1}{2}ft^2$

$$10 = \tfrac{1}{2}f \times 2^2 = 2f$$
$$f = 5$$
$$s = \tfrac{1}{2}f \times 3^2$$
$$s = \tfrac{5}{2} \times 9 = 22\tfrac{1}{2}$$

Ans. $22\frac{1}{2}$ feet.

Example 3.—What is meant when it is said that the acceleration of the velocity of a particle is 10, units being feet and seconds? If the particle were moving at any instant at the rate of $7\frac{1}{2}$ feet a second, after what time would its velocity be quadrupled, and what distance would it describe in that time? [*S. & A.*, 1881.

(1) The acceleration of a particle is 10, units being feet and seconds. By this is meant that the velocity of the body is increased by 10 feet per second in each second, or that in each second an additional velocity of 10 feet per second is given to it.

(2) We have given that the initial velocity at the commencement of any instant is $7\frac{1}{2}$;

$$\therefore V = 7\tfrac{1}{2}$$
$$\text{also } v = 30$$
$$f = 10$$

Therefore we have—

$$v = V + ft$$
$$30 = 7\tfrac{1}{2} + 10$$
$$22\tfrac{1}{2} = 10\,t$$
$$t = 2\tfrac{1}{4} \text{ seconds}$$

(3)

$$s = Vt + \tfrac{1}{2}ft^2$$
$$= 7\tfrac{1}{2} \times 2\tfrac{1}{4} + \tfrac{1}{2} \times 10 \times (2\tfrac{1}{4})^2$$
$$= \tfrac{15}{2} \times \tfrac{9}{4} + \tfrac{1}{2} \times 10 \times \tfrac{81}{16}$$
$$= \frac{135}{8} + \frac{405}{16} = \frac{270 + 405}{16} = \frac{675}{16}$$
$$= 42\tfrac{3}{16}$$

Ans. $42\frac{3}{16}$ feet.

We have said that, supposing the resistance of the air were removed, all bodies, whatever their weights, would fall through the same heights during the same times. This is quite true, and yet there may be some doubt lingering in the mind, arising, it may be, from some such thought as this, "Surely it would be a deal more pleasant to catch a feather which has fallen, say, 40 feet, than a leaden bullet which has fallen through the same height." We must be careful not to confuse the motions of the bodies with the forces producing the motions, or with the effects produced by the motions and forces combined.

So far in our subject we have been dealing more especially with the motions apart from the forces producing the motions, or apart from the effects produced by the motions combined with the forces producing them.

The thought just expressed will be best dealt with under the head of "Momentum;" but before defining this word it will be well to state and to illustrate certain laws upon which rest several questions we have to discuss.

Sir Isaac Newton stated them as follows :—

1. *Every body will remain in its state of rest or of uniform motion in a straight line unless acted on by some force external to itself.*

This law we have already spoken of and illustrated.

2. *Change of motion in a body is proportional to the external force producing that change, and is in the direction due to that external force.*

3. *An action is always opposed by an equal reaction, or the mutual actions of two bodies are always equal and act in opposite directions.*

We will now explain and illustrate the second law.

This speaks of the change of motion in a body and says that—

1. The *amount* of the change depends upon the magnitude of the force producing the change, *i.e.* a force of magnitude 1 lb. being supposed to produce a change of a certain amount in the motion of a body; a force of 2 lbs. will produce a change of twice that amount; a force of 3 lbs. will produce a change of thrice that amount; and so on.

2. The *direction* of the change of motion is due to the direction of the force producing the change.

To illustrate—

1. A person riding in a train going at the rate of 40 miles an hour plays with a ball, which he throws up and catches again. The ball takes, say, one second on its ascent, and, from what we have learnt, the same time, *i.e.* one second, in its descent. During these two seconds of the ball's motion the train, and of course the person, has passed through

$$\frac{\overset{2}{\cancel{4}} \times 176\cancel{0} \times \cancel{3}}{\cancel{6} \times \cancel{60}} \times 2 \text{ feet}$$

$$= \frac{352}{3} \text{ feet} = 117\tfrac{1}{3} \text{ feet}$$

At first sight one would think that the ball would be left behind this distance; but we know this is not so, for the person catches the ball just as though the train had been standing still. How shall we explain this? The ball thrown up is subject to two forces, each of which imparts to it its special change of motion.

(1) That of projection. This will give the ball a vertical

motion, the height ascended being found by the equation $h = \frac{V}{2g}$, V being the velocity of projection.

(2) That of the motion of the train. This will give it a horizontal motion, the distance passed through in the given time being obtained from the equation $s = vt$, where v is the velocity of the train.

Thus we see that the change of motion produced is due to the two external forces acting on the ball, each of which obeys the second law of motion, *i.e.* the ball in its ascent and descent will move, not in a vertical nor in a horizontal direction, but in a direction which is the effect of the two forces combined, the vertical height reached being the same as though the train had not been moving, the horizontal distance traversed being exactly the same as though the ball had moved subject only to the motion of the train.

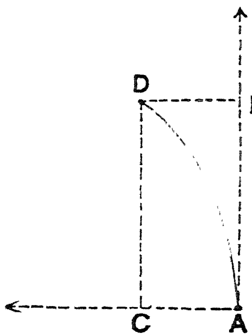


FIG. 61.

As in figure A B showing the vertical height reached, A C the horizontal distance due to the motion of the train, A D the actual path of the ball.

2. A ball is fired from the mouth of a gun against a boat which is moving in a direction at right angles to the gun. The ball passing from one side of the boat to the other will do so in an oblique direction, the amount of obliquity being due to the motion of the boat.

3. A person walking through a shower holds his umbrella aslant, whereas if he stands still he holds it vertical.

4. A ship is moving at the rate of thirty feet per second. A leaden bullet is dropped from the top of a mast. If the ship had been at rest it would have reached a point at the foot of the mast. It reaches the same point when the ship is in motion. This tells us that it has really passed through the horizontal distance traversed by the ship as well as the vertical height of the mast. This horizontal distance is due to the motion of the ship. This example can be worked out in the same way as the first example given.

We can further illustrate this law if we can clearly understand the relations between the force producing change of

motion in a body, the change of motion produced, and the mass of the body acted upon.

As will at once be seen, the change produced by a force of 10 lbs. acting on a mass of 100 lbs. will be very different from the change produced by the same force on a mass of 1 lb.

In considering the masses of bodies in motion we must bear in mind the two parts of the subject being considered—

(1) The rate of motion, *i.e.* the velocity.

(2) The mass of the body in motion.

Speaking of these two subjects together we get what is termed the *momentum* of the moving body, *i.e.* the *quantity of motion* in the moving body. The momentum of a moving body is in proportion to

(1) The velocity of the body ;

(2) The mass of the body ;

and can therefore be measured by the product—

$$M \times V$$

M being the mass of the body
V „ velocity „

This can be illustrated by the first example in this section. The difference between catching the feather and the leaden bullet depends not only on the velocities with which the bodies are moving, but also on their mass.

The steam-hammer is of service because of its momentum, which is measured by the product of its velocity of descent into its mass.

Having defined and illustrated the term *momentum* we will substitute this word for the word *motion* in the statement of the second law, as *quantity of motion* is what is really signified there by the word *motion*.

This will enable us to consider the masses of the bodies moved.

The law will now read thus—

Change of momentum (quantity of motion) in a body is proportional to the external force producing that change, and is in the direction due to that external force.

It will be well here to give some numerical examples of *momentum* and afterwards to deduce certain useful propositions from the second law, using the law as just now stated.

Example 1.—Compare the momenta of two bodies, of masses 10 lbs. and 12 lbs. respectively, moving at the rates of 20 feet per second and 12 miles an hour respectively.

In working out examples on momenta we must be careful to use the same units of mass, time, and space. It will be well to use 1 lb. as the unit of mass, 1 foot and 1 second as the unit of space and time respectively.

Here we have—

$$\begin{aligned} (1)\ M &= 10 \\ V &= 20 \\ \therefore M V &= 10 \times 20 \\ (2)\ M &= 12 \end{aligned}$$

$$V = \frac{\cancel{12}^{\,2} \times 176\cancel{0} \times 3}{6\cancel{0} \times 6\cancel{0}_{\cancel{30}_{10}}} = 17\tfrac{6}{10} = 17\tfrac{3}{5}$$

$$\therefore M V = 12 \times 17\tfrac{3}{5}.$$

$\therefore$ Momentum of first : momentum of second :: $200 : 12 \times 17\frac{3}{5}$
:: $125 : 132$

Example 2.—A steam-hammer weighs 1 cwt., and it falls through 9 feet; another weighs 100 lbs., and falls through 16 feet; compare their momenta.

We have to find the velocity of each at the end of its fall.

This we get from the equation $v^2 = 2gs$.

$$\begin{aligned} (1)\ v^2 &= 64 \times 9 \\ \therefore v &= 24 \\ (2)\ v^2 &= 64 \times 16 \\ \therefore v &= 32 \end{aligned}$$

i.e. the velocity of the 1st is 24 feet per second
" " 2nd is 32 " "

$\therefore$ Momentum of first : momentum of second :: $24 \times 112 : 32 \times 100$;
:: $21 : 25$.

Example 3.—Define momentum. If the mass of a body is 12 lbs., and it is moving at the rate of 10 feet a second, what is the numerical value of its momentum, the units being pounds, feet, and seconds? What would be the numerical value of the momentum if the units were hundredweights, yards, and hours? [*S. & A.*, 1892.

We have defined the momentum of a moving body as the *quantity of motion* in the moving body. It can be measured by the product—

$$\text{Mass} \times \text{velocity}$$

$\therefore$ in the case given we have—

$$\begin{aligned} (1)\ \text{mass} &= 12 \text{ lbs.} \\ \text{velocity} &= 10 \text{ feet per second} \\ \therefore \text{momentum} &= 12 \times 10 = 120 \\ (2)\ \text{Mass} &= 12 \text{ lbs.} = \tfrac{12}{112} \text{ cwt.} \end{aligned}$$

$$\text{velocity} = 10 \text{ feet per second} = \frac{10 \times 60 \times 60}{3} \text{ yards per hour.}$$

∴ momentum (units being hundredweights, yards, and hours)

$$= \frac{\overset{4}{\cancel{12}}}{\underset{\underset{7}{\cancel{28}}}{\cancel{112}}} \times \frac{10 \times 60 \times \overset{15}{\cancel{60}}}{\cancel{3}}$$

$$= \frac{10 \times 60 \times 15}{7} = \frac{9000}{7} = 1285\tfrac{5}{7}$$

Example 4.—The masses of two bodies (P and Q) are in the ratio of 3 to 2; the former is moving at the rate of $7\frac{1}{2}$ miles an hour, the latter at the rate of 200 yards a minute; find the ratio of P's momentum to Q's momentum. [*S. & A.*, 1892.

What is meant by the momentum of a body?

P's momentum : Q's momentum :: mass of P × velocity of P : mass of Q × velocity of Q.

(1) P's mass : Q's mass :: 3 : 2.

∴ If P's mass = 3 m, we have Q's mass = 2 m.

(2) P's velocity = $7\frac{1}{2}$ miles an hour = $\frac{15}{2} \times 1760$ yards an hour

$$= \frac{\cancel{15}}{\cancel{2}} \times \frac{\overset{220}{\cancel{1760}}}{\underset{4}{\cancel{60}}} \text{ yards a minute}$$

= 220 yards a minute

∴ P's momentum : Q's momentum :: $3\,m \times 220 : 2\,m \times 200$
:: $3 \times 22 : 2 \times 20$
:: $3 \times 11 : 20$
:: $33 : 20$

∴ P's momentum : Q's momentum :: 33 : 20

We will now look at the second law of motion as last stated.

The change of momentum in bodies of equal mass will result from the change of velocity, not from the change of mass. This change of velocity will in the cases we have to consider be that of a uniform acceleration due to a constantly acting force, so that taking forces p and P each acting on a body of mass m, and producing accelerations of f and F respectively, by the second law we have—

$$mf : m\mathrm{F} :: p : \mathrm{P}$$

since the forces producing changes of momentum are p and P, and the changes of momentum are $m\,f$ and m F.

Therefore we have in this case the result—

$$f : \mathrm{F} :: p : \mathrm{P}$$

or forces acting on bodies of equal mass are in the ratio of the accelerations produced.

This will be clearer if a numerical example be given.

Suppose two forces each acting on a body whose mass is 20 lbs. produce accelerations of 2 feet per second and 3 feet per second respectively. What is the relation between the two forces?

We have to find the relation between p and P. We know that the accelerations producing the changes of momentum are 2 feet per second and 3 feet per second respectively. Also that the changes of momentum are 20×2 and 20×3.

$$\therefore \text{ we have } 20 \times 2 : 20 \times 3 :: p : P$$
$$2 : 3 :: p : P$$

Changes of momentum in different bodies will result either from the changes of velocities or from changes of masses.

The changes of velocities in the cases we have to consider will be those of uniform accelerations, so that taking two forces, p and P, acting respectively on two bodies of masses, m and M, and producing accelerations f and F respectively, we have by the second law—

$$mf : Mf :: p : P$$

since the forces producing changes of momenta are p and P and the changes of momenta are represented by mf and M F. We have two special cases to consider—

(1) If $f = F$, *i.e.* if equal accelerations are produced, here

$$m : M :: p : P$$

or forces producing the same acceleration in different masses, are proportional to the masses.

(2) If $m = M$ we have—

$$f : F :: p : P$$

as already obtained.

Suppose two forces acting on bodies whose masses are 20 lbs. and 30 lbs. produce accelerations of 2 feet per second and 3 feet per second respectively; what is the relation between the two forces?

We have to find the relation between p and P. The changes of momenta are—

$$20 \times 2 \text{ and } 30 \times 3.$$

Therefore we have—

$$40 : 90 :: p : P,$$
$$i.e.\ 4 : 9 :: p : P.$$

Again, a force of 10 lbs. acts on a body and produces an acceleration of 5 feet per second; another force of 8 lbs. acts on a second body and produces an acceleration of 15 feet per second. Compare the masses of the bodies.

Let the mass of the 1st body be m
" " 2nd " M
Then the change of momentum of the 1st is $m \times 5$
" " " 2nd is $M \times 15$

Therefore our proportion gives us—

$$m \times 5 : M \times 15 :: p : P$$
$$m \times 5 : M \times 15 :: 10 : 8$$
$$m : M \times 3 :: 5 : 4$$
$$\text{or, } 15M = 4m$$
$$i.e.\ m : M :: 15 : 4$$

We have elsewhere defined the "poundal" or "absolute unit of force" as that amount of force which acting for one second on one pound of matter will produce in it a velocity of one foot per second. We will use this definition and deduce certain results from it by means of the second law as stated. Thus, taking the amount of force called the poundal as unity, we are able to use this unit to measure forces.

Thus, considering a force, P, acting on a mass, M, and producing an acceleration of f feet per second, and remembering that the poundal, the unit of force, produces in a mass of 1 lb. an acceleration of one foot per second, we have in our proportion, $mf : MF :: p : P$, the following—

$$p = 1, m = 1, f = 1, F =$$

Therefore our proportion becomes—

$$1 : Mf :: 1 : P$$
$$\text{or, } P = Mf$$

or the force P which produces on a body a change of momentum Mf must be Mf times as great as that force (the poundal) which produces in a second body a change of momentum represented by unity.

We have given previously the equation—

$$W = Mg.$$

This will be found to be the same as the one just stated, the force acting on the mass in this case being the body's weight, which produces in each second an acceleration of g feet per second. Take the two equations—

$$P = Mf \quad \ldots\ldots\ldots\ldots (1)$$
$$W = Mg \quad \ldots\ldots\ldots\ldots (2)$$

Eliminate M by dividing (1) by (2).

We get—
$$\frac{P}{W} = \frac{f}{g}$$
or, $P : W :: f : g$.

This ratio can be expressed thus—

Power producing motion : whole mass in motion :: acceleration produced : g.

It will assist the student to remember this ratio as just stated. It expresses the fact that, if a power P act on a mass W during one second and cause an acceleration, the acceleration produced in each second is f, where—

$$= \frac{P}{W} g$$

g being the acceleration due to the force of gravity.

The application of these principles will be best seen by a number of numerical examples.

Example 1.—A force of 5 lbs. is made to move a body of mass 50 lbs.; what is the acceleration produced?

Using the proportion—

Power producing motion	:	whole mass in motion	$:: f : g$
5	:	50	$:: f : 32$
1	:	10	$:: f : 32$

$$10f = 32$$
$$f = 3\tfrac{1}{5}$$

Example 2.—Find the space passed through during 5 seconds by a mass of 12 lbs., which is made to move from rest by a force of 6 lbs.

(1) To find the acceleration f—

Power producing motion	:	whole mass in motion	$:: f : g$
6	:	12	$:: f : 32$
1	:	2	$:: f : 32$

$$2f = 32$$
$$f = 16$$

(2) To find space passed through—

$$s = \tfrac{1}{2}ft^2$$
$$s = \tfrac{1}{2} \times 16 \times 5^2$$
$$= 8 \times 25 = 200$$

Ans. 200 feet.

Example 3.—A body whose mass is 3 lbs. moves in a straight line under the action of a constant force, P. At a certain point it is moving at the rate of 10 feet a second; at a point 100 feet further on it is moving at the rate of 50 feet a second; find P in poundals, and by how much the velocity is increased in each second of its motion. [*S. & A.*, 1893.

To find f, we have—

$v^2 = V^2 + 2fs$, since its velocity is noted at two separate times; first, when its velocity (V) = 10 feet a second; and, secondly, when its velocity (v) is 50 feet a second. Also, space passed through during interval = 100 feet;

$$\therefore\ 50^2 = 10^2 + 2f \times 100$$
$$\therefore\ 2400 = 200f$$

$\therefore f = 12$, *i.e.* its velocity is increased in each second of its motion by 12 feet per second.

Also $P = mf$ where m = mass = 3 lbs.

$$\therefore\ P = 3 \times 12$$
$$\therefore\ P = 36$$

Ans. P = 36 poundals.

Example 4.—The masses of two bodies are in the ratio of 7 : 5, and they move along straight lines under the action of forces P and Q; the velocity of the former body is increased by 12 feet a second in 3 seconds, the velocity of the latter is increased by 1260 yards a minute in half a minute; find the acceleration of the velocities, and the ratio of P to Q.

(1) To find acceleration of first body—

Its velocity is increased by 12 feet a second in 3 seconds

$\therefore$ " " $\frac{12}{3}$ feet a second in 1 second

$\therefore$ its acceleration = 4 feet per second in each second.

(2) To find acceleration of second body.

The velocity is increased by 1260 yards a minute in $\frac{1}{2}$ a minute.

$\therefore$ " " 1260×3 feet a minute in 30 secs

" " $\frac{1260 \times 3}{60}$ feet a second in 30 secs

" " $\frac{1260 \times 3}{60 \times 30}$ feet a second in 1 sec.

its acceleration = $\frac{126\cancel{0} \times \cancel{3}}{60 \times \cancel{3}\cancel{0}}$ feet per second in each second

$= 2\frac{1}{10}$ feet per second in each second.

Next, to find ratio of P to Q—

$P = m_1 f_1$ where m_1 is mass of 1st body
f_1 is acceleration of 1st body.

Similarly $Q = m_2 f_2$

$$\therefore P : Q :: m_1 f_1 : m_2$$
$$\text{but } m_1 : m_2 :: 7 : 5$$
$$\text{also } f_1 = 4$$
$$f_2 = 2\tfrac{1}{10}$$
$$\therefore P : Q :: 7 \times 4 : 5 \times 2\tfrac{1}{10}$$
$$:: 28 : 10\tfrac{1}{2}$$
$$:: 56 : 21$$
$$\therefore P : Q :: 8 : 3$$

Example 5.—Define the British absolute unit of force or poundal. The masses of two bodies are 5 lbs. and 7 lbs. respectively; at any instant the former is moving at the rate of 12 feet a second, the latter at the rate of 900 yards a minute. The former has acquired its velocity by the action of a constant force, P, in 3 seconds, the latter by the action of a constant force, Q, in a quarter of a minute; find the ratio of P to Q, and express each of them in poundals.

Velocity of first body = 12 feet a second, acquired in 3 seconds.

$$\therefore v = f t$$
$$12 = f \times 3$$
$$\therefore f = 4$$

Velocity of second body = 900 yards a minute

$$= \frac{90\cancel{0} \times 3}{6\cancel{0}} \text{ feet a second}$$
$$= 45 \text{ feet a second.}$$

This is acquired in a quarter of a minute, *i.e.* in 15 seconds

$$\therefore 45 = f \times 15$$
$$\therefore f = 3$$

Now, $P = m_1 f_1$ where m_1 f_1 are mass and acceleration respectively of 1st body.

Similarly $Q = m_2 f_2$.

but mass of first body = 5 lbs

$$\therefore P = 5 \times 4$$
$$\textit{i.e. } P = 20 \text{ poundals}$$
$$Q = 3 \times 7$$
$$\textit{i.e. } Q = 21 \text{ poundals}$$
$$\text{and } P : Q :: 20 : 21.$$

EXAMPLES ON CHAPTER X.

1. Write out a definition of "acceleration." How do you make the definition include a retardation? What does an acceleration always result from?

2. Mention the units of time and distance usually employed in questions on motion. State the meaning of each letter in the formula $s = Vt \pm \frac{1}{2}ft^2$. Why must you not say that s is the "distance passed over"?

3. If in the solutions of questions on the previous formula s were found to equal $+ 24$, 0, or $- 20$, what would be your explanation of these results?

4. Explain clearly the meaning of the formula $V = v \pm ft$. If $V = 0$ in a result, how would you explain this? Also explain if $V = - 12$.

5. Represent graphically the distance passed over by a body moving for 5 secs. with a uniform velocity of 7 ft. per second; also by a body starting from rest and moving for 3 secs. under a constant increase of velocity per second of 7 ft. per second.

6. If the measure of an acceleration is given by 5 when the units are feet and seconds, what will it be when the units are yards and minutes?

7. Express an acceleration in feet per second per second, which is represented by 720 yds. per minute per minute.

8. If the velocity of a body is increased uniformly in each second of its motion by 32 ft. a second, by how many feet a second is its velocity increased in one minute? [*S. & A.*

9. The velocity of a moving body is increased by 12,000 yds. a minute in each minute of its motion; by how many feet a second is its velocity increased in each second of its motion? [*S. & A.*

10. The velocity of a body is increased uniformly in each minute of its motion by 66,000 yds. a minute; by how many feet a second is its velocity increased in each second? If the acceleration of a body's velocity, due to the action of a certain force, is 55 in feet and seconds, what is it in yards and minutes? [*S. & A.*

11. The velocity of a body is increased uniformly in each second by 20 ft. a second; by how many yards a minute will its velocity be increased in one minute? [*S. & A.*

12. If the constant acceleration of a body's velocity be 5 ft.-second units, what is the exact meaning of this statement? and what velocity would the body have at the end of 10 secs., supposing it not to have an initial velocity? [*S. & A.*

13. Suppose the previous body to be moving in such a way that it would pass over 20 ft. in 2 secs. if left alone. If now it undergoes a negative acceleration of 2 ft. per second per second, after how long will its velocity be 4, and when will it come to rest? What circumstances in nature would render possible a motion such as the above?

14. A body moves from a state of rest. At the end of 2 secs. its velocity is 10 ft. per second; end of another 2 it is 20; at the end of another 2, 30. Is the body moving under a constant acceleration? What would cause such an acceleration?

15. What must you know about an acceleration in order that it may be clearly defined?

16. The velocity of a train is known to have been increasing uniformly. At one o'clock its velocity was 12 miles per hour; at ten past one it was 36 miles per hour; what was its velocity at seven and a half past one o'clock? [*S. & A.*

17. State exactly what is meant when the velocity of a body is said to be uniformly accelerated? The acceleration of a body's velocity is denoted by 5, the units being feet and seconds; what fact is expressed by this number 5? [*S. & A.*

18. How is the acceleration measured when constant? A body, whose velocity is known to be uniformly accelerated, is moving at the rate of 100 yds. a minute, and 10 secs. afterwards at the rate of 160 yds. a minute; what is the acceleration of the velocity, and what distance does it describe in those 10 secs.? [*S. & A.*

19. A body moves from rest under a constant acceleration of 5 ft. per second; what distance does it describe in 10 secs.? What distances in the first, second, third, and seventh seconds of its motion?

20. Show that the distances described in successive seconds by a body moving under a uniform acceleration form an arithmetical progression. [*S. & A.*

21. A body has described 50 ft. from rest in 2 secs.; what is the acceleration? How long will it take to pass over the next 150 ft.?

22. A body, moving under the action of a constant force, is 63 ft. from the starting-point at the end of 4 secs.; what is the rate of change of velocity?

23. At the distance of the moon the acceleration of a body falling towards the earth is $\frac{1}{112}$ ft. per second units. If such a body were to fall freely for one hour, what would be its velocity in foot-minute units?

24. A ball is projected along a smooth, flat surface with a velocity of 4 ft. per second, and receives an increase of velocity every second of $1\frac{1}{2}$ ft. per second; find (1) the velocity at the end of 10 secs., and (2) the distance passed over in that time.

25. A body starts from rest and moves with uniform acceleration 18 foot-second units; find the time required to traverse the first, second, and third foot respectively.

26. What is the numerical value of an acceleration which in half a second would produce a velocity which would carry a body over 8 ft. every half-second? [*S. & A.*

27. A body moves over 90 ft. in the fifth second of its motion; if the body started from rest, what is the acceleration in feet per second per second?

28. A body starts with a certain velocity, and in the sixth second moves over 49 ft., and in the eleventh moves over 89 ft.; find the initial velocity and the acceleration.

29. A body starts to move with a velocity of 70 ft. per second, and undergoes a negative acceleration of 10 ft. per second per second; how far will it move before coming to rest?

30. If F denotes the acceleration due to a force acting on a body which moves along a straight line, by what formula can you calculate the velocity the body acquires in a certain time, and the distance it describes in that time, supposing the body to be at rest when the force begins to act upon it? A body known to be acted on by a constant force begins to move from rest; it passes over 55 ft. in a certain 2 secs., and over 77 ft. in the next 2 secs.; what distance did it describe in the first 6 secs. of its motion? [*S. & A.*

31. Two bodies, whose velocities are accelerated in every second by 3 and 5 ft. a second respectively, begin to move towards each other at the same instant, and without any initial velocity. At first they are 1 mile apart; after how many seconds will they meet? [*S. & A.*

32. A particle moves along a straight line, and its velocity undergoes a constant acceleration; write down a formula for the distance it describes in a given time, stating the meaning of each letter in the formula. Apply to the following case: A body moving along a straight line is known to be acted upon by a constant force. At a certain instant it is moving at the rate of 12 ft. a second, and in the next 10 secs. it describes a distance of 470 ft.; what velocity does it gain in each second of its motion? [*S. & A.*

33. A body undergoes a constant retardation of 7 ft. per second per second; how far will it move before its velocity is altered from 19 ft. to 9 ft. per second?

34. A body moves under the action of a constant force, and in 20 ft. its velocity is increased from 5 to 25 ft. per second; find the rate of increase of velocity.

35. If a train is moving at the rate of 45 miles per hour, and at the end of 10 miles is moving at the rate of 60 miles per hour, find the acceleration of its velocity in mile-hour units.

36. What is meant by the momentum of a body? What units are employed? and in what units will the answer be given?

37. One body has a mass of 2 cwts. and moves at the rate of 4 ft. a second; the other has a mass of 21 lbs. and moves at the rate of 120 yds. a minute; find the ratio of the momentum of the former body to that of the latter. [*S. & A.*

38. Two bodies whose masses are 2 lbs. and 3 lbs. respectively are moving at the rate of 21 and 16 ft. per sec.; find the ratio of their momenta. [*S. & A.*

39. The masses of two bodies P and Q are in the ratio of 3 to 2; the former is moving at the rate of $7\frac{1}{2}$ miles an hour, the latter at 200 yds. a min.; find the ratio of P's momentum to Q's momentum. [*S. & A.*

40. Starting with the equation supplied by the 2nd law of motion, viz. $P = mf$, explain each letter and derive the "momentum equation" $Pt = mv$.

41. In the equation $Pt = mv$; if m is expressed in lbs., in what units will P be given? What force, in poundals, acting for 2 secs. on a mass of 32 lbs., will give a velocity of 5 ft. per sec.?

42. The masses of two bodies are 5 lbs. and 7 lbs. respectively; at any instant the former is moving at the rate of 12 ft. per sec. and the latter 900 yds. a min. The former has acquired its velocity by the action of a constant force, P, in 3 secs., the latter by the action of a constant force, Q, in a quarter of a minute; find the ratio of P to Q, and express each of them in poundals. [*S. & A.*

CHAPTER XI.

ENERGY, ETC.

WE have elsewhere defined the term *work*. Bodies are said to possess *energy* when they are able to do work; that is, when they are able to overcome resistance. Thus the water in the

mill-dam is able to do work because of its position; the steam-hammer in its highest position is able to do work because of this position. Such are instances of potential or statical energy, *i.e.* these bodies have power to overcome resistance because of their position.

The fly-wheel in motion is able to do work because of its motion; similarly the rifle-bullet as it is fired from the gun, the train as it rushes along, etc., are said to possess energy because they are able to do work. These are instances of bodies possessing kinetic energy, because they have power to overcome resistance due to their motion.

Let us examine the latter kind of energy.

We will use as an illustration the train weighing 20 tons moving at the rate of 30 miles an hour. The train is capable of doing a certain amount of work. How shall we measure this amount?

The train s velocity is 30 miles an hour

" " $\frac{30 \times 1760 \times 3}{60 \times 60}$ feet per second

" " 44 feet per second.

Suppose that at the moment when it has this velocity the brake is applied, which at once produces a negative acceleration of 4 feet per second. The train will come to rest after passing through a space s obtained from equation

$$v^2 = 2fs$$
$$44^2 = 2 \times 4 \times$$
$$s = 242 \text{ feet}$$

Whilst moving through this space the motion of the train has been against the resistance P of the brake, so that if we knew this resistance P, we should be able to obtain the product $P \times s$, in this case $242 \times P$, which would be the amount done by the train before coming to rest, and therefore the amount of work it was able to do when the brake was applied, *i.e.* we could tell the amount of kinetic energy the train then possessed.

Elsewhere we have obtained the equation—

$$P = Mf$$

where M is the mass of the train.

$$\therefore f = \frac{P}{M}$$

Therefore if this value of f be substituted in the equation—

$$v^2 = 2\,fs$$

$$\text{we have } v^2 = 2 \times \frac{P}{M} \times s$$

$$i.e.\ \tfrac{1}{2}M\,v^2 = Ps$$

Therefore the kinetic energy the train had at the moment of applying the brake was

$$\tfrac{1}{2}Mv^2$$

$$\text{or, remembering } M = \frac{W}{g}$$

$$\text{we have energy} = \tfrac{1}{2}\frac{W}{g} \cdot v^2$$

In this case this equals $\frac{1}{2} \times \dfrac{20 \times 112 \times 20}{32} \times 44^2$

Another term used in the place of kinetic energy is "accumulated work."

The reason why this name is given is because, supposing the body to have started from rest, this same amount of work must have been done upon the body, *i.e.* stored up or accumulated in the body in order to impart to it the given velocity v; or, supposing the body to be moving with the velocity v, it is capable of doing the amount of work before its power to do work becomes zero, *i.e.* before its energy becomes zero, or before it is brought to rest.

Example 1.—A body whose mass is 10 lbs. is moving at the rate of 30 feet a second. What is its kinetic energy or accumulated work in foot-pounds?

Equation to be used is—

$$\begin{aligned}\text{Energy} &= \tfrac{1}{2} \cdot \frac{W}{g} \cdot v^2 \\ &= \tfrac{1}{2} \cdot \tfrac{10}{32} \times (30)^2 \\ &= \frac{10 \times 30 \times 30}{2 \times 32} = \frac{5 \times 15 \times 15}{8} = 140\tfrac{5}{8}\end{aligned}$$

Ans. $140\frac{5}{8}$ foot-pounds.

Example 2.—Define a poundal and a foot-poundal. A body whose mass is 6 lbs. is moving at the rate of 8 feet a second; how many foot-poundals of work can it do against a resistance in virtue of its mass and velocity? If it did 117 foot-poundals of work against a resistance, what would then be its velocity?

[*S. & A.*, 1891.

To find kinetic energy of body—

$$\text{Energy} = \tfrac{1}{2}Mv^2 = \tfrac{1}{2} \times 6 \times 8^2 = 3 \times 64 = 192$$
$$\text{,,} \quad = 192 \text{ foot-poundals.}$$

It can therefore do 192 foot-poundals of work against a resistance.

Suppose it has done 117 foot-poundals of work it has then (192 − 117) foot-poundals of energy remaining = 75 foot-poundals.

Its mass is 6 lbs.; therefore we obtain its velocity at that time from equation—

$$E = \tfrac{1}{2}Mv^2$$
$$\therefore 75 = \tfrac{1}{2} \times 6 \times v^2$$
$$\therefore 25 = v^2$$
$$\therefore v = 5$$

$\therefore$ answer required is velocity = 5 feet per second.

The latter part of the question could have been worked thus—

Work done = change of kinetic energy

$$\therefore 117 = \tfrac{1}{2} \times 6(8^2 - v^2)$$
$$\therefore 39 = 8^2 - v^2$$
$$\therefore v^2 = 25$$
$$\therefore v = 5$$

Ans. 5 feet per second.

Example 3.—A body whose mass is 20 lbs. moves in a straight line against a constant resistance (R); at a certain point it is moving at the rate of 18 feet a second; after moving over 50 feet its velocity is reduced to 10 feet a second; what part of its kinetic energy has it lost? What is the numerical value of R in poundals? [*S. & A.*, 1892.

Let v_1 be velocity at 1st point
,, v_2 ,, ,, 2nd ,,

$$\text{Then kinetic energy at 1st point} = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2} \times 20 \times 18^2$$
$$\text{,, ,, ,, 2nd ,,} = \tfrac{1}{2}mv_2^2 = \tfrac{1}{2} \times 20 \times 10^2$$
$$\therefore \text{kinetic energy lost} = \tfrac{1}{2} \times 20(18^2 - 10^2)$$
$$= 10 \times 28 \times 8$$
$$= 2240$$

Ans. kinetic energy lost = 2240 foot-poundals.

Second part of question.

The kinetic energy lost has been expended in overcoming the resistance R through the given space, therefore the product R × s is equivalent to the kinetic energy expended.

$$\therefore Rs = \tfrac{1}{2}m(v_1^2 - v_2^2)$$

or in the case before us—

$$R \times 50 = \tfrac{1}{2} \times 20(18^2 - 10^2) = 2240$$
$$\therefore R = \tfrac{2240}{50} = 44\tfrac{4}{5}$$

Ans. resistance R = $44\frac{4}{5}$ poundals.

Example 4.—Write down the equation of work and energy for a constant force acting on a particle.

A particle moving from rest is acted on through 250 feet by a force of 9 poundals: find its kinetic energy, and its mass being 5 lbs. find its velocity. [*S. & A.*, 1893.

Its kinetic energy is measured by the product resistance overcome × space through which resistance is overcome = R × *s*.

$$\therefore \text{ in present case energy} = 9 \times 250$$
$$E = 2250 \text{ foot-poundals}$$

also $E = \frac{1}{2}mv^2$ where m = mass of body, v velocity at which it is moving, but in present case m = 5 lbs.

$$\therefore 2250 = \tfrac{1}{2} \times 5 \times v^2$$
$$\tfrac{4500}{5} = v^2$$
$$900 = v^2$$
$$\therefore v = 30$$

Ans. velocity = 30 feet per second.

Example 5.—Define kinetic energy. Write down the formula for the kinetic energy of a particle whose mass is m and velocity v.

The mass of a particle is 10 lbs.; at any instant it is moving at the rate of 24 feet a second; it moves against a constant resistance of 4 poundals; what distance would it describe from that instant before coming to rest? [*S. & A.*, 1892.

To find its kinetic energy—

$$E = \tfrac{1}{2}m \times v^2 = \tfrac{1}{2} \times 10 \times 24^2$$
$$\text{also } E = R \times s$$
$$= 4 \times s$$
$$\therefore 4 \times s = \tfrac{1}{2} \times 10 \times 24^2$$
$$\therefore s = \frac{10 \times 24^2}{4 \times 2} = \frac{10 \times 24 \times 24}{8}$$
$$= 10 \times 24 \times 3 = 720$$

Ans. 720 feet.

Examples on Chapter XI.

1. Explain the meaning of each letter in the energy equation, $\frac{1}{2}mv^2 = Rs$. What is the term $\frac{1}{2}mv^2$ called?

2. Explain the terms potential and kinetic energy. In the same body, what is the connection between the two? What is meant by "conservation of energy"?

3. When will your answer be given in foot-poundals, and when in foot-pounds for the term $\frac{1}{2}mv^2$?

4. A body weighing 16 lbs. falls freely from a height of 100 ft.; what is its energy in foot-poundals and foot-pounds when halfway down and when at the bottom?

5. A body 10 lbs. weight is moving with a velocity of 200 ft. per sec.;

find its energy in foot-pounds. If it does 200 ft.-pounds of work, what is then its energy?

6. A mass of 32 lbs. is moving at the rate of 10 ft. per sec. It runs for 50 ft. against a resistance of $\frac{1}{4}$ lb.; what will then be its velocity?

7. How far will a train of 88 tons, moving initially at the rate of 27 miles per hour, go before it comes to rest, the resistances amounting to 40 lbs. per ton?

8. A shot of 100 lbs. moving at the rate of 1600 ft. per sec., strikes a fixed target; how far will the shot penetrate the target, exerting on it an average pressure of 12,000 tons?

9. A cannon ball whose mass is 60 lbs., falls through a vertical height of 400 ft.; what is its energy? With what velocity must such a ball be projected from a cannon to have initially an equal energy?

10. A body weighing 200 lbs. is moved from a state of rest and is found subsequently to be moving at the rate of 12 ft. a sec.; how many units of work must have been done on it by the forces causing motion over and above those expended in resistances? Give the answer in (*a*) kinetic units, (*b*) in foot pounds ($g = 32$). [*S. & A.*

11. A body of 10 lbs. weight is moving at the rate of 50 ft. per sec.; how far will it go against a constant resistance equal to $\frac{1}{20}$ of this weight before being brought to rest? [*S. & A.*

12. Define a poundal and a foot-poundal. A body whose mass is 6 lbs. is moving at the rate of 8 ft. a second; how many foot-poundals of work can it do against a resistance in virtue of its mass and velocity? If it did 117 ft. poundals, what would then be its velocity? [*S. & A.*

13. If the mass of a body is 15 lbs., and its velocity 12 ft. a second; how many foot-poundals of work can it do against a resistance in virtue of its mass and velocity? What name is commonly given to a body's capacity for doing work in the virtue of its mass and velocity? [*S. & A.*

14. A body whose mass is 20 lbs., moves in a straight line against a constant resistance, R; at a certain point it is moving at the rate of 18 ft. a second; after moving over 50 ft. its velocity is reduced to 10 ft. a second, what part of its kinetic energy has it lost? What is the numerical value of R in poundals? [*S. & A.*

15. A particle whose mass is 5 lbs. falls freely through 10 ft.; what kinetic energy (in foot-poundals) does it acquire? If instead of falling freely it were fastened by a fine thread to a mass of 3 lbs., and the thread were placed on a smooth point so that the mass of 5 lbs. has to draw the mass of 3 lbs. up; find the kinetic energy acquired by the mass of 5 lbs. in falling 20 ft. [*S. & A.*

16. The mass of a particle is 10 lbs., at any instant it is moving at the rate of 24 ft. a second. It moves against a constant resistance of 4 poundals; what distance would it describe from that instant before coming to rest? [*S. & A.*

CHAPTER XII.

HYDROSTATICS—INTRODUCTORY.

WE have already divided matter into three classes, taking, as our reason for division, the power the matter under consideration has of retaining its shape and size. Thus we have said—

A solid has a definite size and shape; a liquid has a definite size but not a definite shape; whilst a gas has neither a definite size nor a definite shape.

We can examine matter with regard to other qualities, *e.g.* I try to drive a nail into a piece of wood; I find the wood resists the progress of the nail, and I have to exert force to accomplish my purpose. I drop the same nail into a bucket filled with water or on to the ground. In both these cases the nail experiences very little if any resistance.

The fish swimming in the water appears to do so and to change the direction of its motion with the greatest ease. The balloon passes through the air without much difficulty. The boy runs about in the playground, apparently without knowing that he is moving in an ocean of air, *i.e.* of matter. On the contrary the stonecutter must use a hammer and chisel to bring the rough block to the needed size and shape. I must use a diamond if I wish to cut a piece of glass.

From these examples it will be seen that we can divide matter into two great classes with respect to the ease or difficulty with which their particles may be separated.

Thus solids may be defined as bodies made up of particles of matter which exist in certain fixed positions with regard to each other, and which cannot be displaced from these positions without the action of great force.

Fluids are those bodies which are made up of particles of matter which exist in no fixed positions with regard to each other, and which can be separated by the slightest force.

In the chapters to follow we shall more especially deal with fluids.

This class of bodies includes the two divisions, liquids and gases.

A liquid resembles a gas in that its particles may be easily separated, but differs from a gas in other important respects.

Thus, as we have seen, a liquid is incompressible whilst a

gas is very compressible, *i.e.* liquids resist compression, whilst gases yield to the slightest pressure.

As liquids are incompressible they are also inelastic; gases, on the other hand, being compressible, are elastic. This can be easily proved by the experiment described in the chapter on "Properties of Matter," see Fig. 37. The piston having been forced in some distance, let the pressure exerted by the hand be removed; the piston at once returns to its original position, and thus shows that the air within the cylinder is elastic.

We have the following table—

- MATTER
  - SOLIDS (impenetrable, *i.e.* particles resist separation)
    - Definite size
    - Definite shape
    - Slightly compressible
      - Some elastic
      - Some inelastic
  - FLUIDS (penetrable, *i.e.* particles do not resist separation)
    - LIQUIDS
      - Definite size
      - No definite shape
      - Incompressible
      - Inelastic
    - GASES
      - No definite size
      - No definite shape
      - Very compressible
      - Very elastic

CHAPTER XIII.

THE BRAMAH PRESS.

THE fact that liquids are incompressible is of very great practical value; from this we have the Bramah press. We will now explain the principle of its action. To do this it will be well to use a few simple introductory illustrations.

1. Take the boy's squirt. We need not explain its construction, but just state that the water leaves the squirt with exactly the same amount of force as the boy presses on the piston with his hand. Here the tube is straight and the

pressure is transmitted from one end of the squirt to the other by means of the water.

Imagine the orifice of the squirt to be as large as the piston. Then if the boy push with a force of 1 lb. and the water is to be kept from issuing from the squirt the orifice must be kept closed by a force of 1 lb., which must be exerted in the direction opposite to the boy's pressure. This is similar to the result we should have had if the water had been replaced by some solid, say a piece of wood. If we push the wood at one end with a force of 1 lb. we must push in the opposite direction with an equal force if we wish to produce equilibrium.

2. Take the hydrant tube used for washing the pavement. We have here a new principle: the man using it turns it about in every direction, yet the water issues with the same force; and if it is to be prevented from issuing we must close the orifice by a force acting at right angles to the breadth of the tube and equal in magnitude to the force with which it enters the tube.

This simple illustration teaches us—

(1) The water transmits the pressure as in the case of the squirt.

(2) The tube need not be straight, or the force at one end of the tube need not be in the same straight line with the force which keeps the orifice closed, when we wish to preserve equilibrium.

(3) The force of the water acts at right angles to the breadth of the tube. If the tube be closed by a piston the force would be at right angles to the piston.

3. Let our third illustration be a vessel, a section of which is represented in the figure. Suppose it filled with liquid, say water. Let A, B, C, D, E be cylindrical orifices in which pistons each of 3 inches area fit air-tight. If now a weight of 3 lbs. be placed on A, we shall find that if B, C, D, E are to be kept closed we must apply a force of 3 lbs. to each one in a direction perpendicular to the piston and towards the liquid. This tells us that the pistons B, C, D, E are each acted upon by a force of 3 lbs. in a direction perpendicular to itself and outward from the liquid.

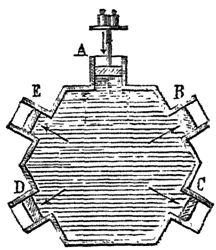


FIG. 62.

Thus we learn that the water inside the vessel has transmitted the force of 3 lbs. acting on A equally in all directions to the pistons B, C, D, E.

4. In this case, instead of pistons A, B, C, D, E all being of equal area, let A be 3 inches in area as before; B be twice the area of A, *i.e.* 6 inches; C be thrice the area of A, *i.e.* 9 inches; D be four times the area of A, *i.e.* 12 inches; and E be five times the area of A, *i.e.* 15 inches. If now 3 lbs. be placed on A as before, we shall find that, to keep the others closed—

B will need a perpendicular pressure inwards of 6 lbs.					
C	"	"	"	"	9 lbs.
D	"	"	"	"	12 lbs.
E	"	"	"	"	15 lbs.

From this illustration we learn the following, which is known as Pascal's Law—

Pressure exerted anywhere upon a mass of liquid is transmitted undiminished in all directions and acts with the same force on all equal surfaces and in a direction at right angles to those surfaces.

We are now in a position to understand the principle of the Bramah press.

The essential part of it consists of two cylinders which are connected one with the other by means of a pipe. The cylinders are fitted with air-tight pistons, as in the figure. Let A be the area of the large one, a that of the smaller. If the piston of the smaller be pushed down with a pressure p, then as the pressure p is exerted on the area a the pressure exerted on the area A will be $\frac{A \times p}{a}$. This will be better understood if we take a numerical example. Suppose A to be 10 square inches and a 2 square inches; then if p equal 5 lbs. we have on every 2 square inches of the larger piston a pressure of 5 lbs., that is, on the whole area a pressure of $\frac{10}{2} \times 5$ lbs. $= 25$ lbs.

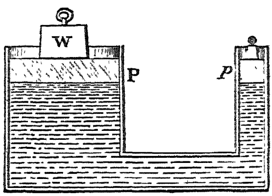


FIG. 63.

The parts of the Bramah press will be best made out from the figure, B, the large cylinder, carrying its air-tight piston P, which carries the table upon which is placed the material to be compressed; A, the smaller cylinder, carrying its air-tight

piston p, worked by means of the lever M; K, the connecting pipe; n, a leathern collar.

These are the most important parts, besides which it has a safety valve, screws, etc., which need not be discussed here.

The collar n is too important a part to be omitted without a slight notice. Its purpose is to make the piston P air-tight

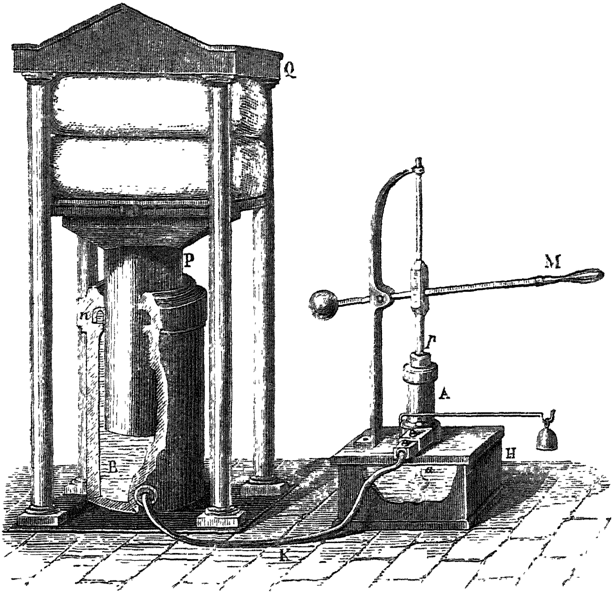


Fig. 64

when subjected to great pressures. It is made of a circular piece of leather saturated with oil out of the middle of which a circular hole is cut. This is doubled so as to present a convex and a concave surface. It is fitted into a groove cut in the cylinder in such a manner that the concave surface is downwards towards the water. As the water is forced upwards into the collar the sides of the collar are thus made to clasp more tightly the cylinder and the piston, and so as the pressure increases the piston becomes more air-tight.

We will work out one or two examples on this machine.

1. A man presses on the handle of the lever of a Bramah

press with a force of 32 lbs. The radii of the two cylinders are 2 inches and 8 inches respectively. The length of the arms of the lever are 4 feet and 1 foot respectively. What force can the man bring to bear upon the matter to be compressed?

(1) To find the pressure exerted on the smaller piston we must remember that the forces are exerted on the lever as in the figure.

$$\therefore \text{ we have } 32 \times 48 = 12 \times F.$$

$$F = \frac{32 \times 48}{12} = 128$$

Pressure on smaller piston = 128 lbs.

(2) Area of smaller piston $= \pi \times r^2 = \pi \times 2^2 = 4\pi$
where r = radius of smaller piston = 2 inches

$\therefore$ pressure on 4π sq. in. = 128 lbs.

$$\therefore \quad \text{,,} \quad 1 \text{ sq. in.} = \frac{128}{4\pi} \text{ lbs.}$$

$$= \frac{32}{\pi} \text{ lbs.}$$

(3) Area of larger piston $= \pi R^2 = \pi \times 8^2 = \pi \times 64$
where R = radius of large piston = 8 inches

$\therefore$ pressure on 64π square inches

$$= \frac{64\pi \times 32 \text{ lbs.}}{\pi} = 64 \times 32 \text{ lbs.}$$

$$= 2048 \text{ lbs.}$$

2. Taking a more general case. A man exerts a pressure of p lbs. on the handle of the pump, which has arms a inches and b inches long respectively.

The area of the larger cylinder is A square inches, and that

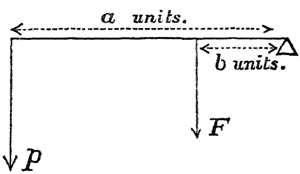


FIG. 65.

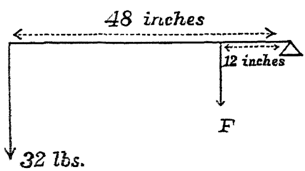


FIG. 66.

of the smaller B square inches; find the pressure on the larger piston.

(1) By the principle of moments $p \times a = F \times b$.

$$\therefore F = \frac{p \times a}{b} \text{ lbs.}$$

(2) The pressure on B square inches $= \frac{p \times a}{b}$ lbs.

Therefore on 1 square inch the pressure $= \frac{p \times a}{B \times b}$ lbs.

(3) Therefore pressure on A square inches is—

$$p \times \frac{a}{b} \times \frac{A}{B} \text{ lbs.}$$

The hydrostatic paradox is an example of the equal transmission of pressure.

It is thus stated: "Any quantity of liquid, however small, can be made to support any weight however large."

It can be illustrated by means of an apparatus like the one represented by the figure.

This consists of a long narrow pipe A B, communicating with a vessel C D, into which is fitted an air-tight movable top. Let C D be loaded with a weight W. If now water be poured into the pipe at A the water will enter the vessel C D, and thus W will be raised. The weight which can be thus raised can be determined by calculating the relative areas of a horizontal section of A B and of C D. Thus, suppose the area of such a section of C D is 100 times that of A B; then the weight W which can be raised will be 100 times the weight of the column of water in the pipe.

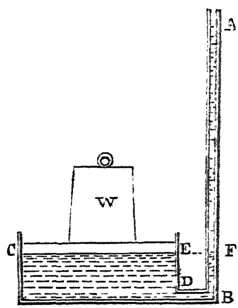


FIG. 67.

Example 1.—A barrel is filled with liquid and closed by a bung, the area of which is 6 square inches, and which can withstand a total pressure of 33 lbs. What pressure per square inch must be applied to the liquid to drive out the bung?

The total pressure on the bung is 33 lbs.
The total area of the bung is 6 square inches.
$\therefore$ the pressure on each square inch $= \frac{33}{6}$ lbs. $= 5\frac{1}{2}$ lbs.
$\therefore$ pressure applied per square inch must be greater than $5\frac{1}{2}$ lbs.

Example 2.—In a Bramah press the diameters of the cylinders are as 3 : 112. What pressure on the smaller piston will support a weight of 10 tons acting on the larger? And what weight can be raised by the larger if the pressure on the smaller be 18 lbs.?

Pressure on smaller piston : weight on larger :: Area of smaller : area of larger.

But areas of cylinders are as $\pi r^2 : \pi R^2$.

Where r and R are radii of small and large cylinders respectively.

$$\therefore \text{ areas are as } \pi\left(\frac{d}{2}\right)^2 : \pi\left(\frac{D}{2}\right)^2$$

Where d and D are diameters.

$$\therefore \text{ areas are as } \pi(\tfrac{3}{2})^2 : \pi(\tfrac{112}{2})^2$$
$$\therefore \text{ areas are as } \tfrac{9}{4} : 3136.$$
$$\therefore \text{ pressure on smaller piston : weight on larger} :: \tfrac{9}{4} : 3136$$
$$\therefore \text{ pressure on smaller : 10 tons} :: \tfrac{9}{4} : 3136.$$

$$\begin{aligned} \therefore \text{ pressure required} &= \frac{10 \times 9}{4 \times 3136} \text{ tons.} \\ \text{,,} \quad \text{,,} &= \frac{10 \times 9 \times 2240}{4 \times 3136} \text{ lbs.} \\ \text{,,} \quad \text{,,} &= \frac{10 \times 9 \times 20}{4 \times 28} \text{ lbs.} \\ \text{,,} \quad \text{,,} &= \frac{10 \times 9 \times 5}{28} \text{ lbs.} \\ \text{,,} \quad \text{,,} &= 16\tfrac{1}{14} \text{ lbs.} \end{aligned}$$

Second part of Question.

Pressure on smaller : weight on larger :: $\frac{9}{4}$: 3136.
Pressure on smaller in this case = 18 lbs.
$\therefore$ 18 lbs. : weight on larger :: $\frac{9}{4}$: 3136.

$$\begin{aligned} \therefore \text{ weight required} &= \frac{3136 \times 18 \times 4}{9} \text{ lbs.} \\ \text{,,} \quad \text{,,} &= 3136 \times 8 \text{ lbs.} \\ \text{,,} \quad \text{,,} &= 25{,}088 \text{ lbs.} \\ \text{,,} \quad \text{,,} &= 11 \text{ tons } 4 \text{ cwts.} \end{aligned}$$

Example 3.—In a hydraulic press the radii of the cylinders are 2 inches and 6 feet respectively. The small piston is worked by a lever 2 feet long, the piston being attached 3 inches from the fulcrum. What power must be applied to the end of the lever to raise a girder weighing 14 tons which rests on the larger piston?

Note.—The words "Bramah" and "hydraulic" refer to the same press.

From the figure, by the principle of moments, we have—

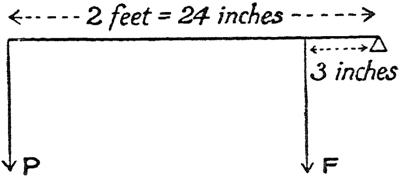


Fig. 63.

$$F \times 3 = P \times 24$$
$$\textit{i.e.}\ F = 8P$$

$$\therefore \text{ pressure on larger piston} = 8P \times \frac{\text{area of larger piston}}{\text{area of smaller piston}}$$

But area of larger piston $= \pi \times (72)^2$ square inches

But area of smaller piston $= \pi \times (2)^2$ „ „

Also pressure on larger piston $= 14$ tons $= 14 \times 2240$ lbs.

$$\therefore \text{ we have } 14 \times 2240 \text{ lbs.} = \frac{8 \times P \times \pi \times (72)^2}{\pi \times (2)^2}$$
$$= \frac{8 \times P \times 72 \times 72}{4} = P \times 2 \times 72 \times 72$$

$$\therefore P = \text{power to be applied} = \frac{\cancel{14}^{\,7} \times \cancel{2240}^{\,140}}{\cancel{2} \times \cancel{72}_{\,18} \times \cancel{72}_{\,18}} \text{ lbs.} = \frac{7 \times 35}{9 \times 9} = 3\tfrac{2}{81} \text{ lbs.}$$

Examples on Chapter XIII.

1. What are the three states in which matter exists? Mention a body which commonly occurs in all three.

2. What is a fluid? Name some common fluids. Why should water and nitrogen be classed together as fluids?

3. What important feature distinguishes water (a liquid) from a pile of sand (a solid)?

4. What is a perfect fluid? Is water a perfect fluid? How would you show that there is a certain amount of friction between the particles of the liquid in a cup of tea when in motion?

5. How does it follow from the definition of a fluid that the pressure of water on the sides of the containing vessel is exerted at right angles to the surface?

6. Mention all the points you can in which a liquid resembles a gas—and in which it does not resemble a gas.

7. If you pour water from a bottle into a tea-cup, why does the water readily change its shape from that of the bottle to that of the tea-cup? Do you know any liquids that would not readily do so? What property of liquids does this exemplify?

8. Write out the statement called "Pascal's law."

9. State the law of transmission of pressure through a fluid. Give a simple illustration of the statement that fluids at rest offer no sensible resistance to force tangentially applied. [*S. & A.*

10. Why does a diver hold his open hands above his head while diving? Why is it very dangerous to fall on the chest when diving?

11. Give some common illustrations of Pascal's law—*i.e.* the pressure is transmitted equally in all directions?

12. What is meant by the (1) "pressure on a point"; (2) "pressure at a point."

13. Write out a proof that the surface of a liquid at rest is horizontal. Is the surface of a lake really level?

14. What do you know about the pressure at any point within a fluid?

15. Describe the hydrostatic bellows, or the hydrostatic paradox. Why is it possible to burst a barrel of water by inserting a thin tube at the top and pouring water down it?

16. A vessel full of water is fitted with a tight cork. How is it that a slight blow on the cork may be sufficient to burst the vessel?

17. Mention the principle on which the Bramah press depends. Describe very briefly its principal parts. Which parts are absolutely essential? [*S. & A.*

18. If the diameters of the cylinders in a Bramah press be 3 ins. and 112 ins. respectfully, what pressure on the smaller piston will support 10 tons on the larger?

19. In a Bramah press if the area of the small piston be 13 sq. ins. and that of the larger 260, what pressure will 18 lbs. exerted on the smaller support of the larger?

20. If the area of the pistons are as 5 : 168, what pressure must be exerted on the smaller to raise a weight of 6 tons?

21. In a tub full of water there are two holes filled by tightly fitting pistons. One piston has an area of $\frac{1}{2}$ sq. in. to which a pressure of 10 lbs. is applied. The other has an area of 3 sq. in. What force must be applied to this in order that it may not move?

22. In a hydraulic press the radii of the cylinders are 2 ins. and 6 ft. respectively. The small piston is worked by a lever of the second kind 2 ft. long, the piston being attached 3 ins. from the fulcrum. What power must be applied to the end of the lever to raise a girder weighing 14 tons which rests on the larger piston?

23. If the diameter of the smaller plunger is $\frac{3}{4}$ in., and that of the larger plunger or ram 15 ins., and the arms of the lever handle 2 ins. and 2 ft. respectively, what force must be applied to the end of the long arm of the lever to make the ram raise a weight of 10 tons? [*S. & A.*

CHAPTER XIV.

PRESSURE OF A FLUID AGAINST A PLANE AREA.

As a liquid is made up of a number of particles, each particle being acted upon by the force of gravity, we can at once state that liquids have weight. From this fact we can deduce the form of the surface of a liquid at rest where the area of the surface is small: this will be horizontal, for, as every particle is acted on by the same force, if we could imagine the surface being other than horizontal, say that of an inclined plane, we have the case of a particle resting on an inclined plane acted on by two forces only, seeing that there is no friction between the particles of liquid. These two forces are—

1. the weight of the particle.
2. the reaction of the plane.

And, as these forces act as in the case of a smooth inclined plane, we have as their resultant a force which causes the particle to move down the plane. As this is true of every particle, the inclined plane cannot be the true surface. It will be seen that the only true surface must be the one at right angles to the force of gravity acting on the liquid.

In the case of a large area, say that of a lake, we have the surface still at right angles, at every point, to the force of gravity, giving in this case a surface to some extent curved, seeing that the direction of the force of gravity at one point will not be parallel to the direction at another point some distance off; but these directions will converge to the centre of the earth.

We have then that the surface of a liquid contained in a vessel is horizontal: this is true no matter what the shape of the vessel may be.

Let the vessel be such as represented in the figure.

Here we have one vessel consisting of arms A, B, C, D, all connected with each other by one pipe, $m\,n$. We see that the water stands at the same height in each arm. This principle underlies the saying that "water always finds its own level."

It is illustrated by the spirit-level and also by the fountain supplied by the water in the tank placed at the top of the house, the height of the fountain depending upon the height of the surface of water in the tank.

This is the principle by which the water supply of large

towns is maintained; the water is stored in large reservoirs or dams, which are constructed on high pieces of land outside the towns; the water then flows down through the pipes to the places where it is needed, *i.e.* it is pulled down by its own weight or is acted upon by the force of gravity.

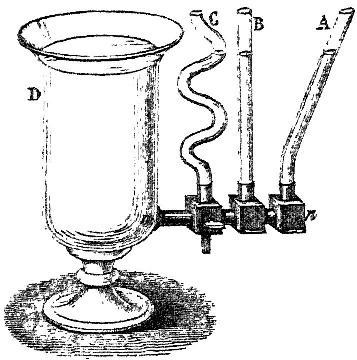


FIG. 69.

As liquids have weight, it will be seen, on considering the liquid contained in a vessel, the lower we descend the greater must be the pressure on each successive layer.

Thus, take the layer *a b*; this is acted on not only by its own weight but also by the weight of the liquid above it; whilst the layer *c d* is acted on by the same weight as *a b*, together with the additional weight of the column *a b c d*.

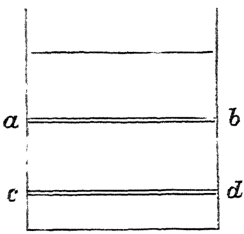


FIG. 70.

Thus the pressure on the bottom of a vessel is equal to the weight of a column of the liquid found by multiplying the area pressed by the depth of the centre of gravity of the area pressed below the surface of the liquid.

We must be careful to understand this point, and to see clearly that the pressure on the area pressed depends not on the quantity of liquid in the vessel but on the depth of the liquid in the vessel.

We can best make this clear by first restating the principle that the liquid exerts a pressure on the sides of the vessel in a direction at right angles to the sides. In the second place let us consider three cases.

1. The vessel having vertical sides. This case will present no difficulty, as clearly here the pressure on the base is at once found by the rule given.

2. The vessel having sides slanting outwards. Here the

pressure on the base is equal to the weight of a column of the liquid whose base equals the base A C of the vessel, and whose height equals A B. To understand this, consider a point O in the side of the vessel. This point is acted on by a force F in the direction O F; but, as the side prevents motion, we have at once a reaction F^1, equal and opposite to F. This reaction is equivalent to the two forces O M, O N. This will be true at every point at the sides of the vessel. The horizontal forces thus obtained will occur in pairs, which will thus balance each other, while the vertical forces N show that part of the weight of the liquid is supported by the base of the vessel, the other part being supported by the slanting sides of the vessel.

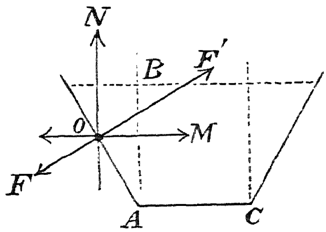


FIG. 71.

3. The vessel having sides slanting inwards. The reasoning in the second case applies here, but the force N acts in the opposite direction, viz. downwards, giving as before the pressure on the base equal to the weight of the column of the liquid, having the area A C pressed for base, and the height equal to B D.

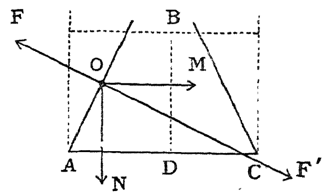


FIG. 72.

The pressure on the sides of a vessel will be found by remembering that the pressure increases with the depth and acts at right angles to the surface. It is found by the rule before stated, viz. if A denote the area pressed and H the depth of the centre of gravity of A below the surface of the liquid, then the pressure is equal to the weight of a column of the liquid whose base is A and whose height is H.

Example 1.—A cubical tank whose edge is 6 feet long is filled with water. Find the pressure on each surface.

(1) A cubic foot of water weighs 1000 ozs.
2) The pressure on the base.

Here $A = 6 \times 6$ square feet
also $H = 6$ feet

Therefore weight of column of water which equals pressure on base $= 6 \times 6 \times 6 \times 1000$ ozs.

(3) The pressure on one side.

Here $A = 6 \times 6$ square feet
also $H = 3$ feet

Therefore weight of column $= 6 \times 6 \times 3 \times 1000$ ozs.
Therefore pressure on each side $= 6 \times 6 \times 3 \times 1000$ ozs.

Example 2.—A cylindrical box whose height is 6 feet and diameter 1 foot is filled with water and held with its axis vertical. Find the pressure on each end and the pressure on the whole curved surface.

(1) Pressure on bottom.

Here $A = \pi \times r^2$ (where r = radius) square feet
$= \pi \times (\frac{1}{2})^2$ square feet $= \frac{22}{7} \times \frac{1}{4}$ square feet
also $H = 6$ feet

Therefore column of water whose weight equals required pressure weighs $\frac{22}{7} \times \frac{1}{4} \times 6 \times 1000$ ozs.

(2) Pressure on top.

Here $A = \frac{22}{7} \times \frac{1}{4}$ square feet as before;
also $H = 0$

Therefore column of water has height $= 0$.
Therefore pressure in this case $= 0$.

(3) On curved surface.

Here $A = 2\pi r \times h$ square feet $= 2 \times \frac{22}{7} \times \frac{1}{2} \times 6$ square feet where h is length of cylinder;
also $H = 3$ feet

Therefore column of water whose weight is equal to required pressure weighs $\frac{22}{7} \times 6 \times 3 \times 1000$ ozs.

Example 3.—The same cylindrical box filled as in Example 2 is held with its axis horizontal. Find the pressure on each end and on the whole curved surface.

(1) Pressure on each end.

Here $A = \pi r^2$ square feet $= \frac{22}{7} \times \frac{1}{4}$ square feet;
also $H = \frac{1}{2}$ feet

Therefore column of water giving required pressure weighs

$$\frac{22}{7} \times \frac{1}{4} \times \frac{1}{2} \times 1000 \text{ ozs.}$$

(2) Pressure on whole curved surface.

Here $A = 2\pi r \times h$ square feet as before
$= \frac{22}{7} \times 6$ square feet
also $H = \frac{1}{2}$ foot

Therefore column of water giving required pressure weighs $\frac{22}{7} \times 6 \times \frac{1}{2} \times 1000$ ozs.

Example 4.—A vessel in the form of a pyramid 6 feet high has a square base the side of which is 4 feet. It is placed on a table and filled with water. Find the pressure on the base and the pressure on the table supposing the vessel to weigh 1 lb.

(1) To find pressure on base of vessel.

Pressure required equals weight of a column of water whose base is area pressed, A, and whose height, H, is depth of centre of gravity of A below surface of water.

Here $A = 4 \times 4$ square feet
$H = 6$ feet

$\therefore$ content of column $= 4 \times 4 \times 6$ cubic feet

But 1 cubic foot of water weighs 1000 ozs.

$\therefore$ pressure required $= 4 \times 4 \times 6 \times 1000$ ozs. $= 96{,}000$ ozs.
$= 6000$ lbs.

(2) To find pressure on table.

This is merely to find weight of water in vessel, and then add to this the weight of the vessel itself. Solid content of pyramid $= \frac{1}{3}$ solid content of column already found.

$$\therefore \text{weight of water in vessel} = \frac{6000 \text{ lbs.}}{3} = 2000 \text{ lbs.}$$

$\therefore$ pressure required $= 2000$ lbs. $+ 1$ lb. $= 2001$ lbs. $= 96{,}000$ ozs.

Example 5.—How is the pressure at any point within a liquid measured? A circle, whose radius is one foot, is described on a vertical wall of a reservoir; the surface of the water is $2\frac{1}{2}$ feet above the centre of the circle; find the ratio of the fluid-pressure at the highest point to the fluid-pressure at the lowest point of the area. Find also the magnitude of the fluid-pressure on the area of the circle. (A cubic foot of water weighs 1000 ozs.) [*S. & A.*, 1892.

(1) To find ratio of pressure at highest point to the fluid-pressure at the lowest point.

Pressure is measured by area pressed $\times$ depth of centre of gravity below surface of water.

$\therefore$ pressure at highest point : pressure at lowest point :: $A \times 1\frac{1}{2}$: $A \times 3\frac{1}{2}$

$\therefore$ pressure at highest point : pressure at lowest point :: 3 : 7

(2) To find the magnitude of fluid-pressure on the area of circle.

$$\text{Pressure required} = \pi \times (1^2) \times 2\tfrac{1}{2} \times 1000 \text{ ozs.}$$
$$\text{where } \pi \times (1^2) = \text{area pressed}$$
$$2\tfrac{1}{2} \text{ feet} = \text{depth of centre of gravity below surface.}$$
$$\therefore \text{pressure required} = \tfrac{22}{7} \times \tfrac{5}{2} \times 1000 \text{ ozs.} = 7857\tfrac{1}{7} \text{ ozs.}$$

Example 6.—A cube, each of whose edges is 2 feet long, stands on one face on the bottom of a vessel containing water 4 feet deep; find the pressure of the water on one of the upright faces of the cube, assuming that a cubic foot of water weighs 1000 ozs.

[*S. & A.*, 1885.

Pressure required is the weight of a column of water whose base is equal to the area pressed, and whose height is equal to the depth of the centre of gravity of the area pressed below the surface of the water.

$$\begin{aligned} \text{Area pressed} &= 2 \times 2 \text{ square feet} \\ \text{Depth of centre of gravity} &= 3 \text{ feet} \\ \therefore \text{solid content of column} &= 2 \times 2 \times 3 \text{ cubic feet} \\ \therefore \text{weight of column} &= 2 \times 2 \times 3 \times 1000 \text{ ozs.} \\ \therefore \text{pressure required} &= 2 \times 2 \times 3 \times 1000 \text{ ozs.} \\ &= 12{,}000 \text{ ozs.} \\ &= 750 \text{ lbs.} \end{aligned}$$

Resultant Pressure.—I take an ordinary balance and find the two scale-pans are equal in weight. I now make an arrangement by means of which one pan hangs suspended in a vessel of water whilst the other is outside the vessel. I now find a great difference, apparently in the weights of the two pans; the one outside seems to be much heavier than the other, as shown by its descending whilst the other ascends. Why is this? Let us examine the forces acting on the pans in both cases. First, each pan is acted on by its own weight and the tension in the chains which uphold it. These forces being equal on both sides of the beam, we have the beam in a state of equilibrium. In the second instance, the pan in the water is not only acted on by its own weight vertically downwards and the tension in its support as before, but also a third force, namely, the resultant of the pressures of the water upon it; this force can be shown to act in a direction vertically upwards, and therefore the weight of the pan seems less than before. This resultant pressure must be clearly understood.

We have seen that surfaces in contact with a liquid are subject to pressures which are perpendicular to the surface. This is true, no matter what the shapes of the surfaces may be, and no matter whether they are the sides of the vessel containing the liquid or are the surfaces of bodies partially or totally immersed in the liquid.

We will deal with resultant pressure under three chief heads—

1. When the surfaces pressed are the sides of the vessel containing the liquid.

2. When the surfaces pressed are those of bodies partially immersed in the liquid.

3. When the surfaces pressed are those of bodies wholly immersed in the liquid.

1. Surfaces being the sides of the vessel containing the liquid.

To find the resultant pressure in this case will be an easy problem if we can resolve all the pressure acting into two, viz.

(1) Those which act in a horizontal direction.

(2) „ „ „ vertical „

For example. To find the resultant pressure on the sides of a cubical box, say, of one foot edge, which is filled with water.

We can deal with the six sides of the box in pairs, viz. two pairs of vertical sides and the top and bottom of the box.

The pressures on the vertical sides will be in equilibrium, since these pressures act in directions perpendicular to the sides of the vessel, and therefore the pressure on one vertical face will be equal to and will act in the opposite direction to the pressure on the opposite face.

The pressure on the top will be zero, since the depth of the centre of gravity of the surface pressed below the surface of the liquid is zero.

Therefore the resultant pressure is that on the bottom of the box, which can be found in the usual way.

The method then to adopt is to resolve all the pressures acting into two, viz.

(i.) The resultant of the horizontal pressures.

(ii.) „ „ „ vertical „

If these be compounded we have the resultant pressure.

2. Surfaces pressed being those of bodies partially immersed. Take as an example a cork, a raft, or a plank floating in the water. If we pursue the same course as in the last case we shall find that the horizontal pressures are in equilibrium, and we have left a vertical upward pressure, the magnitude and point of application of which will at once be found if we remember that the body is in equilibrium and examine the forces acting on the body. These are—

(1) The weight of the body acting vertically downwards from its centre of gravity.

(2) The horizontal pressures which are in equilibrium.

(3) The upward vertical pressure due to the pressure of the water.

Therefore the forces acting on the body are (1) and (3). These are in equilibrium, and therefore they must be equal and must act in opposite directions in one straight line. Therefore the resultant pressure is an upward vertical pressure equal in magnitude to the weight of the water displaced and acting at a point directly below the centre of gravity of the body.

3. Surfaces pressed being those of bodies wholly immersed. This case will present a little more difficulty than the preceding. Take an easy case, say that of a cube suspended in water with its sides vertical by means of a thread. As in the previous case, the horizontal pressures are in equilibrium. The forces then acting on the body which we have to consider are—

(1) The weight of the cube vertically downwards.

(2) The tension in the thread.

(3) The pressure on the top of the cube vertically downwards.

(4) The pressure on the bottom of the cube vertically upwards.

The resultant of these forces will be an upward pressure, which is equal to the weight of a column of the liquid equal in bulk to the cube, *i.e.* equal to the water displaced, which is the resultant pressure required.

Let us now take a more difficult case, viz. a body of any shape. To help us in our consideration, let us suppose that the body is a part of the liquid itself, which has become solid; it will remain suspended in the liquid. By taking the same steps as before and resolving the pressure on every part of the surface in those acting horizontally and vertically, we find that those acting horizontally are in equilibrium, as there is no motion in any horizontal direction; whilst the vertical pressure is in equilibrium with the weight of the body itself, which in this case is the weight of a part of the liquid equal in bulk to the liquid displaced acting from the centre of gravity of the liquid displaced. This point is called the *centre of displacement.*

The whole of this section may be summed up by stating the principles of Archimedes.

A body partially or wholly immersed in a liquid is subject to a pressure acting vertically upwards from the centre of displacement, and which is equal in magnitude to the weight of the liquid displaced.

If we wish to make our statement more clearly explain the experiment at the head of this section, it will read thus—

A body partially or wholly immersed in a liquid appears to lose a portion of its weight equal to the weight of the liquid displaced

Examples on Resultant Pressure. Example 1.—A rectangular box has a partition 10 inches high and 8 inches broad. On one side is water to the depth of 4 inches, and on the other side alcohol to the height of 8 inches. Find the resultant pressure on the partition if a cubic foot of alcohol weighs 800 ozs.

(1) Find pressure of water on one side.
(2) Find pressure of alcohol on other side.

The resultant pressure required will be the difference of these two, as they each act horizontally to the surface of the partition.

(1) Pressure of water $= 8 \times 4 \times 2 \times \frac{1000}{1728}$ ozs.

As area pressed $= 8 \times 4$ square inches.

Depth of centre of gravity of surface pressed below surface of water $= 2$ inches.

$$\therefore \text{ pressure of water} = \frac{64 \times 1000}{1728} \text{ ozs.} = \frac{1000}{27} \text{ ozs.}$$

(2) Pressure of alcohol $= 8 \times 8 \times 4 \times \frac{800}{1728}$ ozs.

As area pressed $= 8 \times 8$ square inches.

Depth of centre of gravity of surface pressed below surface of alcohol $= 4$ inches.

$\therefore$ Pressure of water $= \frac{3200}{27}$ ozs.

$$\therefore \text{ resultant pressure required} = \frac{3200 - 1000}{27} \text{ ozs. on side of alcohol}$$
$$= \frac{2200}{27} \text{ ozs.} = 81\tfrac{13}{27} \text{ ozs.}$$

Example 2.—State how to find the fluid-pressure on a body partly immersed. Suppose the plane of the paper to be vertical; draw a square A B C D, take A E a third of A B, and D F a third of the parallel side D C, and draw a line of indefinite length through E F; let that line represent the surface of the water, and let the square represent a cube (whose edge is a foot long) held in it, with A D under water; find the resultant of the fluid-pressure on the cube. If now we suppose the cube turned round E, so that D comes into the surface of the water, find the resultant pressure in this case and show it in the diagram. [*S. & A.*, 1892.

Taking the first case given here, cube is placed as in figure.

Following out the reasoning of the text, we have the case of a body partially immersed.

Consider the forces acting on the body. These are—

(1) Weight of body vertically downwards from centre of gravity of body.

(2) Horizontal pressures which are in equilibrium.

(3) The vertical pressure due to pressure of water.

Forces (1) and (3) are in equilibrium.

Therefore the resultant pressure is an upward vertical pressure equal in magnitude to the weight of the water displaced, acting at a point directly below the centre of gravity of body.

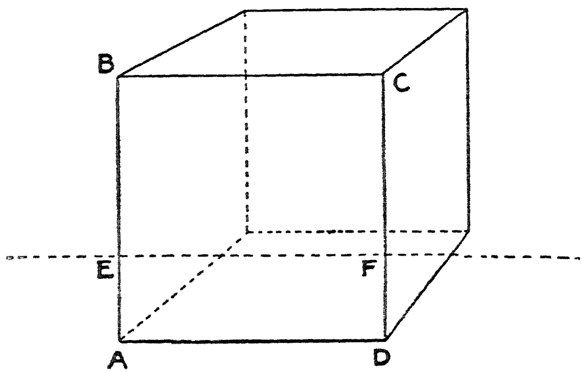


FIG. 73.

Now volume of cube under water = $\frac{1}{3}$ vol. of cube = $\frac{1}{3}$ cubic ft.

$\therefore$ water displaced = $\frac{1}{3}$ cubic ft.

$\therefore$ weight of water displaced = $\frac{1}{3} \times 1000$ ozs.

$\therefore$ resultant pressure required = $\frac{1000}{3}$ ozs. vertically upwards acting from centre of displacement, *i.e.* from the centre of gravity of water displaced.

Second case.—Body displaced as in figure. Dotted figure showing new position. Reasoning as in the last case, we find that again the resultant pressure is an upward vertical pressure equal in magnitude to the weight of the water displaced, acting from its centre of displacement, *i.e.* from the centre of gravity of water displaced.

Now here the volume of the cube immersed = $\frac{1}{6}$ cubic ft.

$\therefore$ upward vertical pressure = $\frac{1000}{6}$ ozs. = $166\frac{2}{3}$ ozs.

This resultant pressure acts from G_2 centre of displacement.

It will be well to notice in this case the effect of the forces acting, which will be different from the effect in the first case.

In the first case we have W, the weight of the cube acting vertically downwards from G, its centre of gravity, and the resultant

pressure of the water acting vertically upwards from G_1, the centre of displacement. Now G and G_1 are in the same vertical line, and we have equilibrium. In the second case we have W acting as before, but the new resultant pressure acting from G_2, a new centre

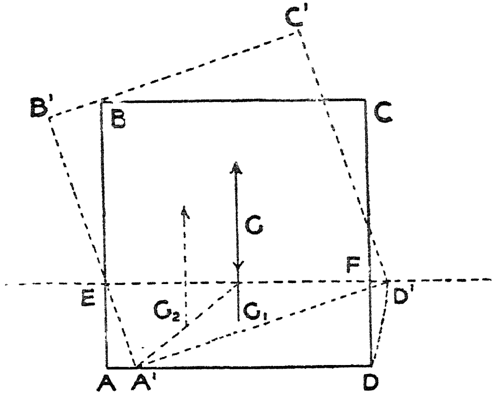


FIG. 74.

of displacement, which is not vertically below G. We have, therefore, two forces not in the same straight line acting in opposite directions. The effect will therefore be to bring the cube back into its original position.

Example 3.—A body whose volume is 5 cubic feet hangs by a string suspended in water which completely covers it. What force exerted along the string is needed to support the body, 1 cubic foot of which weighs 1200 ozs.? Here the forces acting horizontally are in equilibrium. The vertical pressure is in equilibrium with the weight of the body itself, which is the weight of a part of the water equal in bulk to the liquid displaced.

Now the body displaces 5 cubic feet of water.

$\therefore$ weight of water displaced $= 5 \times 1000$ ozs.

The vertical pressure $= 5000$ ozs.

But weight of body $= 5 \times 1200 = 6000$ ozs.

$\therefore$ force required to be exerted along the string

$= 6000$ ozs. $- 5000$ ozs.

$= 1000$ ozs.

$= \frac{1000}{16}$ lbs. $= 62\frac{1}{2}$ lbs.

Centre of Pressure.—The sides of a vessel filled with a liquid, or any surface in contact with a liquid, are subject at every point to the pressure of the liquid. This pressure acts at

right angles to the surface. Thus, taking a plane horizontal surface, the pressure here at every point is vertical. If the surface be vertical the pressure at every point is horizontal. If the surface be oblique the pressure is still at right angles to the surface.

Therefore in every case of plane surfaces pressed by a liquid we have a number of parallel forces acting on the surface, these parallel forces being the pressure at every point of the surface.

If now we find the centre of these parallel forces, we have a point which is called the *centre of pressure* of the surface, *i.e.* we have found the point at which the resultant of these parallel forces may be supposed to act.

If the surface be horizontal this point is the same as the centre of gravity of the surface. This needs no proof, as it will at once come to the mind of the student who recognizes the similarity between the centres of pressure and gravity.

But in cases of vertical and oblique surfaces the centre of pressure is not so easily found. To help us in our determinations we must remember that the pressures are parallel, yet in these cases they are not equal forces. If these pressures were equal the centres of pressure and gravity would coincide, but, as we have already stated, the pressures increase with the depth of the liquid.

For example, to find the centre of pressure of a vertical line drawn on the side of a vessel in contact with water. We know that the pressures are parallel and horizontal, and that they increase proportionally with the depth of the water. Since these pressures increase in this manner, we at once notice the similarity between finding the centre of pressure of this vertical line and the centre of gravity of a triangle. For if lines be drawn across the triangle parallel to the base, we know that the length of these lines increases proportionally with their distance from the apex of the triangle, and each line is proportional to the force of gravity acting at its point of section with the line joining the apex with the mid-point of the base.

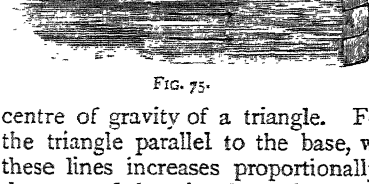


FIG. 75.

The centre of pressure of the vertical line is therefore in the same relative position as the centre of gravity of the triangle, *i.e.* it is at a distance from the surface equal to two-thirds the length of the vertical line.

The same reasoning gives us the position of the centre of pressure of a vertical rectangle, the position being at a point two-thirds from the surface of the vertical line which bisects the rectangle.

In the same manner we get the position of the centre of pressure of a triangle, which has one angle in the surface, and so placed that the line from this angle to the mid-point of the base is vertical. This position is in this line at a distance from the surface equal to three-quarters of this line. The result we obtain by remembering the position of the centre of gravity of the pyramid.

Suppose this triangle were inverted so that the base lies in the surface. In this case the centre of pressure lies in the vertical line bisecting the triangle at a distance from the surface equal to half of this line. The student will be easily able to see the reason for this statement.

We must be careful to remember that the centre of pressure is a *point*, and not a *force*.

Position of Centre of Pressure in a few simple cases.

1. Straight line, at point two-thirds of line from surface.
2. Rectangle, at point two-thirds of line from surface line bisecting both edges of rectangle.

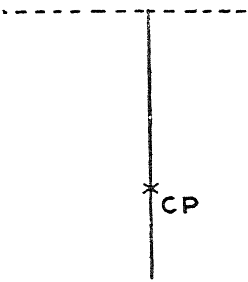


FIG. 76.

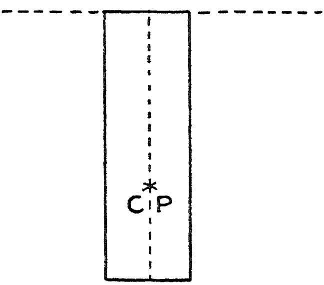


FIG. 77.

3. Triangle with base in surface of water at point bisecting line joining middle point of base to apex.

4. Triangle with apex in water at points three-fourths distance of line from apex to middle point of base.

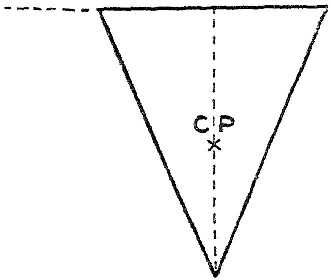


Fig. 78.

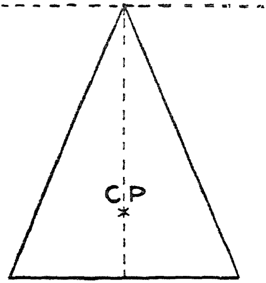


Fig. 79.

Example 1.—What is the centre of pressure of a fluid area? If the area pressed is a rectangle with one edge on the surface of the fluid, where is the centre of pressure? A rectangular hole A B C D, whose lower side C D is horizontal, is made in a side of a reservoir and is closed by a door whose plane is vertical; the door can turn freely round a hinge coinciding with C D; calculate the force that must be applied to A B to keep the door shut, assuming that A B is 1 foot and A D 12 feet long, and that the water rises to the level of A B. [*S. & A.*

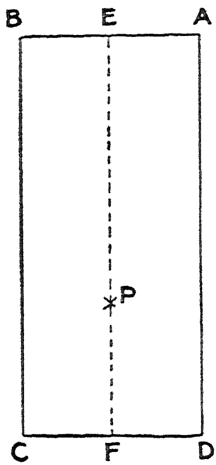


Fig. 80.

We want the total pressure on A B C D. This is measured by—

$$\text{Area of } A\,B\,C\,D \times \frac{A\,D}{2} \times 1000 \text{ ozs.}$$

$$= 12 \times 6 \times 1000 \text{ ozs.} = 72{,}000 \text{ ozs.}$$

This acts from centre of pressure of surface A B C D, *i.e.* from point P where $E\,P = \frac{2}{3}\,E\,F = \frac{2}{3} \times 12$ feet $= 8$ feet.

Therefore, taking moments of forces about point F, we have moment of force required about F is equal to moment of total pressure about F.

$$\therefore \text{force required} \times E\,F = 72{,}000 \text{ ozs.} \times P\,F = 72{,}000 \times 4 \text{ ozs.}$$

$$\therefore \text{force required} = \frac{72{,}000 \times 4 \text{ ozs.}}{12}; \text{ for } E\,F = 12$$

$$\therefore \text{force required} = 24{,}000 \text{ ozs.}$$
$$= \tfrac{24000}{16} \text{ lbs.} = 1500 \text{ lbs.}$$

EXAMPLES ON CHAPTER XIV.

1. What does the pressure at any point within a liquid depend upon? Give an expression for the pressure on a certain area immersed in a liquid.

2. Find the pressures on the sides of a cubical box of 2 ft. side filled with water and closed by a lid. What is it on the lid and bottom?

3. Find the pressure on a slate 12 ins. by 8 ins. completely immersed in water, with the longer sides horizontal and 4 ins. below the surface. What would be this pressure if the liquid were to have a specific gravity of 1·25?

4. What is the pressure on the same slate when held with one longer side on the surface and the other 4 ins. below the surface?

5. What do you mean by the "whole pressure" on a surface immersed? Find the whole pressure on a rectangular surface 6 ft. by 8 ft. immersed vertically in water, the 6 ft. edge being 2 ft. below the surface and parallel to it.

6. Find the whole pressure in the above case if the liquid have a specific gravity of 1·25 and the lower 6-ft. edge is 6 ft. below the surface.

7. Show that if a cubical box be filled with olive oil (sp. gr. ·915) the whole pressures on sides and bottom are equal to three times the weight of the oil.

8. A square pyramid the length of whose base is 3 ins. and height 4 ins. is filled with water; find the pressure on the base.

9. A cylinder-base 12 ins. radius, height 5 ft., is half filled with water and half with oil (·915); find the pressure on the base.

10. A vertical cylinder 3 ft. high and radius of base 1 ft. is filled with water and closed by a heavy lid resting on the water and weighing 3 lbs.; find the pressure on the curved surface of the cylinder.

11. A vertical cylinder 5 ft. high and 1 ft. diameter is filled with a liquid of specific gravity 1·5 and closed by a piston weighing 4 lbs.; what effect has the piston on the whole pressure? Find the pressure on the curved surface.

12. How is the pressure at any point within a liquid measured? A circle whose radius is 1 ft. is described on a vertical wall of a reservoir, the surface of the water is $2\frac{1}{2}$ ft. above the centre of the circle; find the ratio of the fluid-pressure at the highest point to the fluid-pressure at the lowest point of the area. Find also the magnitude of the resultant fluid-pressure on the area of the circle. [*S. & A.*

13. The water is 15 ft. deep on one side of a floodgate 20 ft. wide, and 10 ft. on the other side; find the whole pressures on the sides, and the difference of the pressures.

14. State the rule which enables us to determine the amount of pressure exerted by a fluid against a plane area. A reservoir has one of its walls vertical, a circle 1 yd. in radius is described on the wall. When the water just covers the circle, what is the amount of the pressure exerted by the water on that portion of the wall within the circle? [*S. & A.*

15. A cube, each of whose edges is 2 ft. long, stands on one face on the bottom of a vessel containing water 2 ft. deep; find the pressure of the water on one of the upright faces of the cube. [*S. & A.*

16. A vessel is partly filled with water, and then oil is poured in till it forms a layer 6 ins. deep; find the pressure per square inch due to the weight of the liquids at a point 8·5 ins. below the upper surface of the oil,

assuming the specific gravity of oil to be ·92 and the weight of a cubic inch of water 252 grs. [*S. & A.*

17. A cube, the edge of which is 3 ft., is filled with water; find the pressure on the base and sides. If it were filled with sulphuric acid (1·84) calculate the new pressures.

18. Find the pressure on an equilateral triangle just immersed vertically in water (*a*) when one side coincides with the surface of the water, (*b*) when the vertex is at the surface and the base is parallel to it. Side of the triangle 4 ft.

19. A cylinder 5 ft. long and with a radius of 7 ins. is filled with water; find the pressure on the ends and concave surface (*a*) when axis is vertical, (*b*) when horizontal.

20. A square pyramid 6 ft. high is 4 ft. along the base. It is placed on a table and filled with water; find the pressure on the base and the pressure on the table, supposing the vessel to weight 1 lb. How do you account for the difference?

21. A town is supplied with water from a reservoir, the surface of which is 1000 ft. above the level of the sea. A tap in one of the houses in the town has an area of 4 sq. ins. and is 980 ft. above sea-level; find the pressure of the water on the tap.

22. A barrel 8 ft. high is filled with water. A small tube $\frac{1}{4}$ in. bore is tightly inserted on the top and filled with water to a height of 4 ft.; find the pressure per square inch on the base of cask.

23. Two equal small areas (A and B) are marked on the side of a reservoir at different depths below the surface of the water; what is the ratio of the pressure of the water on A to its pressure on B? The pressure on A is four times the pressure on B, but if water is drawn off so that the surface of the water in the reservoir falls 1 ft., the pressure on A is now nine times that on B. At what depth were A and B below the surface in the first instance? [*S. & A.*

24. Is the "whole pressure" the same as "resultant pressure"? Point out the difference in as many ways as you can.

25. How do you find the resultant vertical pressure on a body immersed? What is the name given to this principle?

26. Prove that the resultant pressure on a solid immersed in a fluid acts vertically, and determine its magnitude.

27. State how to find the resultant fluid-pressure on a body partly immersed. Suppose the plane of the paper to be vertical. Draw a square A B C D, take A E, a third of A B and D F, a third of the parallel side D C, and draw a line of indefinite length through E and F: let that line represent the surface of the water, and let the square represent a cube (1 ft. side) held in it, with A D under water; find the resultant fluid pressure on the cube. If now we suppose the cube turned round E, so that D comes into the surface of the water, find the resultant fluid-pressure in this case and show it in the diagram. [*S. & A.*

28. A piece of cork is held under water by a string. What is the tension on the string?

29. A rectangular box has a partition 10 ins. high and 8 ins. broad, on one side is water to the depth of 4 ins., and on the other side alcohol to a height of 8 ins.; find the resultant pressure on the partition if a cubit foot of alcohol weighs 800 ozs.

30. Carefully define "pressure at a point," "mean pressure," "whole pressure," "resultant pressure," and "centre of pressure."

31. Where is the centre of pressure in the following cases: (*a*) Rectangle immersed vertically, one end on the surface; (*b*) triangle immersed vertically, base on surface; (*c*) ditto apex in surface and base parallel to surface?

32. A rectangular hole A B C D, whose lower side C D is horizontal, is made in a side of a reservoir and is closed by a door whose plane is vertical. The door can turn round a hinge coinciding with C D. Calculate the force that must be applied to A B to keep the door shut, assuming that A B is 1 ft. and A D 12 ft. long, and that the water rises to level of A B. [*S. & A.*

33. State the rule for finding the magnitude of the resultant pressure of a liquid on a plane area immersed in it. A square whose side is 8 ft. long has its plane vertical and its upper edge on the surface of the water in which it is immersed; find the magnitude of the resultant pressure on one face of it. If the square were fixed and the surface of the water were raised a foot, what would now be the magnitude of the resultant pressure?

34. A body whose volume is 5 cub. ft., hangs by a string suspended in water which completely covers it; what force exerted along the string is needed to support the body, 1 cub. ft. of which weighs 1200 ozs.?

35. One side of a reservoir has a slope of 12 vertical to 5 horizontal; what is the whole amount of the pressure of the water against 50 ft. of its length, when the depth of the water is 12 ft.? [*S. & A.*

36. A rectangular reservoir is full of water, its length and depth being respectively 10 ft. and 5 ft., a diagonal line is drawn along one of the vertical sides, dividing it into two triangles; find the magnitude of the resultant fluid-pressures on each of these triangles. Why is the answer independent of the width of the reservoir? [*S. & A.*

CHAPTER XV.

EQUILIBRIUM OF FLOATING BODIES—METACENTRE.

FROM the preceding chapter we learn the two conditions of equilibrium of a floating body. They are—

1. The weight of the body must equal the weight of the fluid displaced.

2. The directions of the weight of the body and of the upward vertical pressure of the liquid must be in the same straight line.

This subject requires a little more attention than we have already given it, as it is an essential point to be considered in the construction of ships, boats, etc.

We will take the two conditions separately—

1. The weight of the body must equal the weight of the liquid displaced.

This condition determines the depth to which the floating body will sink in the water. *E.g.* a certain volume of cork will not sink to so great a depth as the same volume of oak, since the weight of the cork is less than the weight of the oak.

2. The centre of gravity of the body and the centre of gravity of the displaced water must lie in the same vertical line.

It will be seen that the vertical line here referred to corresponds to the vertical line dropped from the centre of gravity of a body resting on a horizontal plane.

The stability of equilibrium will therefore depend upon the fulfilment of this second condition.

The equilibrium of vessels, etc., at sea should be stable, since they are subject to the action of the waves, tides, etc., and are consequently being continually removed from the position in which the centres of gravity of the body and that of the displaced water are in the same vertical line.

We will first explain the terms *stable* and *unstable* equilibrium, and then determine the conditions for these states.

1. *Stable Equilibrium.*—Suppose the body floating in equilibrium to be slightly displaced from this position. If the forces acting upon it tend to bring it back to its original position the equilibrium is said to be *stable.*

2. *Unstable Equilibrium.*—Suppose the body floating in equilibrium to be slightly displaced from this position. If the forces acting upon it tend to carry it further from its original position of equilibrium its equilibrium is said to be *unstable.*

To find the conditions of stable and unstable equilibrium we must know the forces with their directions which act upon the body.

They are two, viz.—

1. The weight of the body vertically downwards.
2. The pressure of the water vertically upwards.

The first acts from the centre of gravity of the body, the second from the centre of gravity of the water displaced.

The body is in equilibrium when G, the centre of gravity of the body, and H, the centre of gravity of the water displaced, are in one and the same straight line.

Instead of calling the point H the centre of gravity of the water displaced, it is usually called the *centre of buoyancy.*

If the body be displaced slightly we have the forces acting as in the figure. The moment of these forces will cause it

to rotate either towards its original position or away from it. Upon the action of this moment depends the stability of equilibrium of the body.

Take the case where G is above H. Let the body be dis-

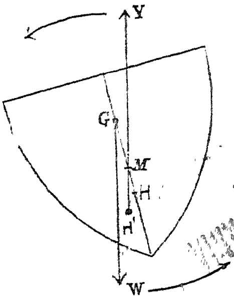


FIG. 81.

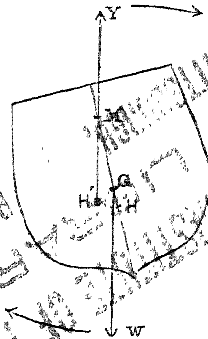


FIG. 82.

placed as in Fig. 81. Then H', the new position which the centre of the water displaced takes up, is the point from which the water acts.

We have therefore the forces—

1. W, the weight of the body, acting from G, tending to carry the body in the direction of the arrowhead, away from the position of equilibrium.

2. Y, the pressure of the water, acting from H' in the direction H' Y, where H' Y cuts H G in the point M. This force also tends to carry the body away from the position of equilibrium. Notice our result in this case—

(1) The equilibrium is unstable.
(2) The point M is below G.

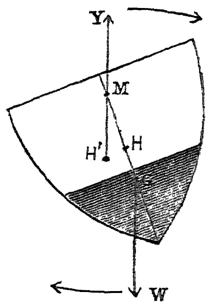


FIG. 83.

Take now a second case where G is above H. Let the body be displaced as in Fig. 82.

Here we have—

1. W, the weight of the body, acting from G as before, but in this case this force tends to restore the body to its original position.

2. Y, the pressure of the water, acting from point H' in

the direction H′ Y, where H′ Y cuts H G in the point M as before; but again this force tends to restore the body to its original position.

Notice our result in this case—

(1) The equilibrium is stable.

(2) The point M is above G.

Take now, as the third case, the point G below H. Here we shall find—

(1) The equilibrium is stable.

(2) The point M is always above G.

On examining our results in the three cases we find that the condition of equilibrium depends upon the relative positions of the points M and G.

If M is below G the body is in unstable equilibrium.

If M is above G the body is in stable equilibrium.

The point M is called the *metacentre* of the body. It may be defined as the point of intersection of the two lines drawn vertically from the centres of displaced water, first in the position of equilibrium, secondly after a slight displace ment.

From what has been said we learn that G must be below M if the equilibrium is to be stable. This tells us that the lower down the centre of gravity of the floating body is, the more stable is its equilibrium. This will explain why some hydrometers are loaded at the bottom end, why ballast is necessary, why it is dangerous to overload the deck of a vessel, etc.

From what has been said we shall at once be able to compare the results now obtained with those in the "states of equilibrium" of a body, and we can tabulate them thus—

For Small Displacements of a Floating Body the Equilibrium is—		
Stable ...	When metacentre is above the c. g. of the body.	*i.e.* when point of intersection of the two lines drawn vertically from centres of displaced water (1) in position of equilibrium, (2) after slight displacement, is above c. g.
Unstable	When metacentre is below the c. g. of the body.	*i.e.* when point of intersection of the two lines, etc., is below c. g. of the body.
Neutral ...	When metacentre and c. g. of body coincide.	*i.e.* when point of intersection of the two lines, etc., coincides with c. g. of body.

It is thus seen that the stability of a floating body depends upon the position of the point M; that is, upon the position of the metacentre relative to the centre of gravity of the body.

Two or three examples will be useful.

1. The metacentre of a sphere partially immersed. Here the metacentre coincides with the centre of the sphere. For

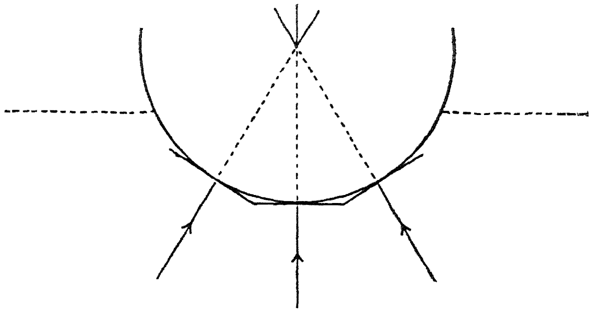


FIG. 84.

the pressure of the liquid on every part of the spherical surface is perpendicular to the surface; it therefore acts along the radius—as this is true when the body is slightly displaced, and the resultant pressure of the liquid must be, in all positions of the floating sphere, a vertical force passing through its centre. Therefore in the position of equilibrium the centre of gravity lies in the vertical line through the centre of the sphere, therefore the centre of the sphere is the metacentre.

In this case the centre of gravity of the body and the metacentre coincide. Therefore the equilibrium is neutral. Take a case where the centre of gravity of the sphere does not coincide with the geometrical centre of the sphere, *e.g.* a loaded sphere, or a sphere containing a cavity. By previous work we know that the only position in which the sphere will remain at rest is when the centre of gravity of the body is vertically below its geometrical centre, and we have the same reasoning as before.

2. The metacentre of a cylinder. A thin disc of wood floats flat on the water that, is, with the axis of the cylinder formed by the disc vertical; the stump of a tree floats with its axis horizontal. In general it is found that when the ratio of the radius of the cylinder to its height exceeds a given quantity,

the position of stable equilibrium is when the axis is horizontal; but when this ratio is less than the given quantity, the position of stable equilibrium is when the axis is horizontal.

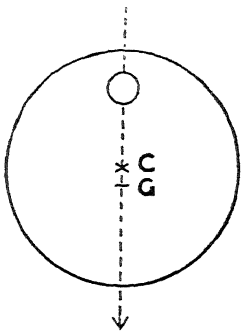


FIG. 85.

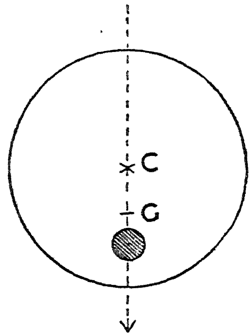


FIG. 86.

To determine the position M of the metacentre the following formula is used. We can only explain the symbols, the proof of the formula is beyond the present work—

$$H\,M = \frac{A\,K^2}{V}$$

where H M = the height of M above the centre of displacement H in the position of equilibrium.

V = volume of liquid displaced by the body

A = area of horizontal section of the body at the surface of the liquid

K = radius of gyration of this area about the axis in this plane about which the body is displaced.

This formula is true for all floating bodies partially immersed.

In the case of the cylinder, suppose—

h = height of cylinder

s = specific gravity of material of which the cylinder is composed

r = area of cross-section of cylinder; this cross-section, being taken at right angles to the height, must be a circle.

Our formula can be much simplified, for V can be replaced

by $\pi r^2 \times h \times s$ = solid content of cylinder × specific gravity of material.

A can be replaced by πr^2 = area of circle forming cross-section.

Also the value of K^2 in this case $= \frac{r^2}{4}$

We have then in place of $H\,M = \frac{A\,K^2}{V}$

$$\text{the formula } H\,M = \frac{\pi r^2 \times \frac{r^2}{4}}{\pi r^2 \times hs}$$

$$\therefore H\,M = \frac{r^2}{4\,hs}$$

$$\text{or } 4\,H\,M \times hs = r^2$$

from which H M and the position of M can be determined.

Though the proof of the formula given for finding the position of M is beyond us at present, it will be well to give the meaning of the expression *radius of gyration.* Suppose a body to be made up of a large number of particles. If we multiply the mass of each particle by the square of its perpendicular distance from a given line or axis and then obtain the sum of all these products, we obtain a result which is called the *moment of inertia* of the body with respect to the axis used. Now the *radius of gyration* is that distance from the axis at which the whole mass of the body may be considered to be collected without producing any change in the result known as the *moment of inertia.*

Examples on Metacentre of Cylinder.—A uniform cylinder of material whose specific gravity is $\frac{5}{6}$ floats in water in stable equilibrium with its axis vertical. Show that the radius of its cross-section is greater then $\frac{\sqrt{10}}{6}$ of its height.

Using formula obtained—

$$4\,H\,M \times h \times \tfrac{5}{6} = r^2$$

$$H\,M = \frac{r^2 \times 6}{h \times 20} = \frac{r^2 \times 3}{h \times 10} \quad \ldots\ldots (1)$$

It has already been shown that if the equilibrium is to be stable the point M must be above the point G,

$$\therefore \text{ H M must be greater than H G}$$

We have therefore to determine the relative values of H M and H G in terms of the length of the cylinder and radius of cross-section.

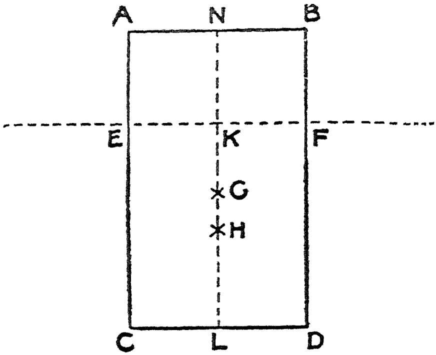


FIG. 87.

Now K L $= \frac{5h}{6}$, as the specific gravity of the material is $\frac{5}{6}$

$\therefore$ H L = K H $= \frac{1}{2} \times \frac{5h}{6} = \frac{5h}{12}$

also G L $= \frac{h}{2}$

$\therefore$ H G = G L − H L $= \frac{h}{2} - \frac{5h}{12} = \frac{h}{12}$ .... (2)

But H M $= \frac{3r^2}{10h}$ from (1)

Therefore the equilibrium is stable if H M is greater than H G

"	"	"	"	$\frac{3r^2}{10h}$	"	"	$\frac{h}{12}$
"	"	"	"	$3r^2$	"	"	$\frac{10h^2}{12}$
"	"	"	"	r^2	"	"	$\frac{10h^2}{36}$
"	"	"	"	r	"		$\frac{1}{6}\sqrt{10h}$

Example 2.—A cylinder floats in water with its axis vertical; find the value of H G, and show that when $s = \cdot 7$ the condition for stable equilibrium is that the ratio of the radius of the cross-section to the height of the cylinder is greater than $\frac{\sqrt{42}}{10}$

Using the figure of the last example, we have—

$$\text{K L} = hs$$
$$\therefore \text{H L} = \text{K H} = \tfrac{1}{2}hs$$
$$\text{also G L} = \frac{h}{2}$$
$$\therefore \text{H G} = \text{G L} - \text{H L} = \frac{h}{2} - \tfrac{1}{2}hs = \frac{h}{2}(1 - s)$$
$$\therefore \text{H G} = \frac{h(1 - s)}{2}$$

for stable equilibrium M must be above G,

$\therefore$ H M is greater than H G; but by formula given H M $= \dfrac{r^2}{4hs}$

$$\therefore \frac{r^2}{4hs} \text{ must be greater than } \frac{h(1 - s)}{2}; \text{ but } s = \cdot 7$$
$$\therefore \frac{r^2}{2\cdot 8 \times h} \quad \text{,,} \quad \text{,,} \quad \frac{h(\cdot 3)}{2}$$
$$\therefore r^2 \quad \text{,,} \quad \text{,,} \quad \frac{h^2 \times \cdot 3 \times 2\cdot 8}{2}$$
$$\therefore r^2 \quad \text{,,} \quad \text{,,} \quad h^2 \times \cdot 42$$
$$\therefore r \quad \text{,,} \quad \text{,,} \quad \frac{\sqrt{42}}{10}h$$

The determination of the position of the metacentre of a cylinder floating partially immersed with its axis horizontal follows at once from the reason given for the case of the sphere floating partially immersed. The position of the metacentre coincides with that of the centre of gravity.

Examples on Chapter XV.

1. Write out the two conditions that must be fulfilled in order that a body may float in a liquid in perfect equilibrium.

2. What is the centre of buoyancy? What must be the relative position of the centres of gravity and buoyancy in order that the floating body may be in (1) stable equilibrium (2) unstable equilibrium? Can you always determine from the relative position of these two points whether the equilibrium is stable or unstable?

3. What is the metacentre of a floating body? Why should you expect a ship sinking in still water to right itself before going down? [*S. & A.*

4. How should the metacentre and centre of gravity be arranged in building a ship, in order that the ship may float in safety?

5. A cylindrical log of wood 18 ins. long floats with its axis vertical in water. To what depth will it be immersed if its specific gravity is 0·6, *i.e.* if a cubic foot of wood weigh 600 ozs.?

6. A cube of oak 970 ozs. to cubic foot, floats partly in sea water (1028 ozs. to cubic foot) and partly in olive oil (915 ozs. ditto). The edge of cube being 6 ins., find what part is immersed in the oil.

7. A cube of wood (750 ozs. to the cubic foot) floats in water; how much of one of its vertical edges is under water? If its edges are 2 ft. long, what is the magnitude of the resultant fluid-pressure on one of its vertical faces? [*S. & A.*

8. A cylinder is 2 ft. high, and the radius of its base is 3 ft. Its specific gravity is ·7, and it floats with its axis vertical; find (*a*) how much of its axis will be under water, (*b*) the force required to raise it 1 in. [*S. & A.*

9. A body whose specific gravity is ·5, floats in water. If the weight of the body be 1000 ozs, find the number of cubic inches of it above the surface of the fluid.

10. If a body of 3 lbs. weight float with three-quarters of its volume immersed in a fluid, what will be the pressure on the hand which keeps it totally immersed?

CHAPTER XVI.

SPECIFIC GRAVITY.

In the last chapter we said that a certain volume of cork would not sink to so great a depth as the same volume of oak, since the weight of the cork is less than the weight of the oak. Instead of the word *weight* we might have used the term *specific gravity*. This term is one of very frequent use, and the determination of the specific gravity of a substance is of very great importance. We will give a definition of the term and the methods by which specific gravities may be found.

The specific gravity of a body is its relative weight compared with water.

To illustrate. The specific gravity of copper is 8·9, by which is meant that a cubic foot of copper weighs 8·9 times as much as a cubic foot of water, *i.e.* it weighs 8900 ozs.

The specific gravity of sulphur is 1·985, by which is meant that a cubic foot of sulphur weighs 1·985 as much as the weight of a cubic foot of water, *i.e.* it weighs 1985 ozs.

To put this in a form easily to be remembered and to assist calculation, let S be the specific gravity of a substance, V its volume, w the weight of a certain volume of water chosen as a unit.

Then, if W be the weight of the substance, w V will be the

weight of the same volume of water, and therefore by the definition given $\frac{W}{wV} = S$.

Example.—A rod of uniform cross-section 18 inches long weighs 3 ozs.; its specific gravity is 8·8. What fraction of a square inch is the area of its cross-section? The weight of a cubic inch of water may be taken to equal 252 grains. [*S. & A.*, 1881.

Let x = the number of square inches in the section.
Then $18 \times x$ = number of cubic inches of volume of rod = V.

It weighs 3 ozs. $= \frac{3 \times 7000}{16}$ grains = W (taking 7000 grains to be equal to one pound avoirdupois).

$$\therefore \frac{\frac{3 \times 7000}{16}}{252 \times 18 \times x} = 8{\cdot}8$$

$$\frac{3 \times 7000}{252 \times 18 \times 8{\cdot}8 \times 16} = x$$

from which we obtain $x = {\cdot}03288$, *i.e.* the section is ·03288 of a square inch.

Methods of determining specific gravity. We have to find the relation between the weight of a volume of the substance and that of the same volume of water.

This can be done by two weighings–

1. That of the volume of the substance.
2. That of the volume of the water.

This would be a most inconvenient method except in the case of liquids and powders. We have given previously the fact that if a body be totally immersed in a liquid it displaces its own volume of the liquid. This will help us to a second method.

If we have a vessel exactly filled with water and then carefully immerse the body whose specific gravity we are finding, we shall find a certain amount of water overflow. This amount is equal to the volume of the body. If then we weigh the water which has overflowed and also the body we can find the specific gravity needed.

This is also an inconvenient method.

We have stated in the section on resultant pressure that the weight of a body when weighed in water appears to be less by the weight of the water displaced. If then we have the two following—

1. The weight of the body in air, or, better, its weight *in vacuo*, which is its real weight;
2. The weight of the body in water, which is the apparent weight of the body,

we know from the principle stated that the difference of the two must be the weight of the water displaced, *i.e.* if W be the true weight and w the apparent weight, then $W - w$ represents the loss of weight in water, or the weight of the displaced water; therefore the relation which W bears to $W - w$ must be the specific gravity of the body,

$$i.e. \frac{W}{W - w} = S$$

Example.—A piece of copper weighs 12 lbs. in air and 10·65 lbs. when weighed in water. What is its specific gravity?

Here—

$$W = 12$$
$$w = 10{\cdot}65$$
$$\therefore \frac{12}{12 - 10{\cdot}65} = S$$
$$\frac{12}{1{\cdot}35} = S$$
$$\frac{4}{0{\cdot}45} = S$$
$$8{\cdot}9 = S$$

We have then to find methods by which we can weigh bodies in air and in water. We must remember, while doing this, the different behaviour of substances with respect to water; thus we have—

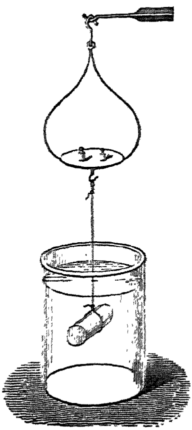


FIG. 88.

Bodies heavier than water.

Bodies lighter than water.

Bodies soluble in water; and also liquids.

The Hydrostatic Balance.—This is an instrument for weighing the body in air and afterwards in water. An ordinary balance provided with a hook below one of the pans will serve to illustrate the method, which consists of two parts. First, we have to find the weight of the body in air; this is done in the ordinary way by weighing the substance in the balance. Now, if the body be heavier than water, in the second place find the

weight of the body in water, by attaching the body by a very fine thread to the hook below the pan and thus suspending it in water; if the weights which have to be removed from the opposite pan to produce equilibrium are noticed we have at once the value of $W - w$, *i.e.* of the weight of the water displaced, and therefore we can use the equation—

$$\frac{W}{W - w} = S$$

Example.—A body when weighed in air is found to weigh 8 ozs., but when weighed in water only 4 ozs. What is its specific gravity?

Here—

$$W = 8$$
$$w = 4$$
$$\therefore W - w = 4$$

Therefore we have—

$$\tfrac{8}{4} = S$$
$$\therefore S = 2$$

This instrument will help us to find the specific gravity of a solid lighter than water. This is done by fastening the light body to another called a *sinker* which is sufficiently heavy to keep both bodies totally immersed; but in this case we shall have to find not only the weight of the two together in water, but also that of the heavy body in water. This can be best illustrated by the following example.

A body weighs 600 grains in air; in order to find its specific gravity by the balance we must attach to it a piece of iron which weighs 800 grains in water. On using the instrument we find the body and iron together weigh 600 grains in water. Find the specific gravity of the body.

Here we have weight of body in air = 600 grains = W.

In the equation previously used $\frac{W}{W - w} = S$. $W - w$ represents the loss of the body's weight in water, but this cannot be obtained in the present instance at once; but we know w, the weight of the iron in water, = 800 grains; also we know $w + w_1$, the weight of the two together in water, = 600 grains. From these we can obtain—

$$W - \{(w + w_1) - w_1\} = W - w$$
$$i.e.\ W = 600 \text{ grains}$$
$$w_1 = 800 \text{ grains}$$
$$w + w_1 = 600 \text{ grains}$$
$$\therefore W - w = 600 - \{600 - 800\}$$
$$= 600 + 200$$
$$= 800$$
$$\therefore \frac{W}{W - w} = \frac{600}{800} = \frac{6}{8} = \cdot 75 = S.$$

From this we see the three steps needed—
1. Find the weight of the body in air = W.
2. Find the weight of the sinker in water = w_1.
3. Find the weight of the two together in water = $w + w_1$.

Having obtained these results, write the equation—

$$\frac{W}{W - w} = S$$

thus—

$$\frac{W}{W - \{(w + w_1) - w\}} = S$$

and then work out from figures obtained.

Nicholson's Hydrometer.—This consists of a hollow cylinder, B, to which is suspended a small loaded cone, C. This cone is loaded so that the metacentre of the whole apparatus when floating in water may be above its centre of gravity, and thus the equilibrium may be stable.

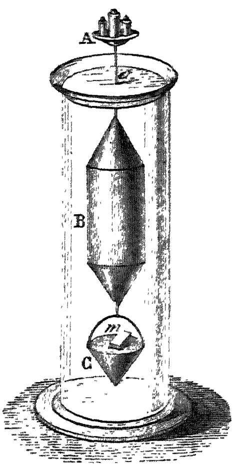


Fig. 89.

To the cylinder is attached a pan, A, by means of a stem, on which a point, *o*, is marked. We will illustrate the use of this instrument to find the specific gravity of iron.

The whole apparatus is placed in water. It floats so that the cylinder and cone are below the surface of the water while the pan is above.

Our first step is to cause the marked point *o* to coincide with the surface of the water. This is done by adding weights. Let us suppose the required weight in the

case we are now considering to be 240 grains; our second step is to find the weight of the iron in air. This we do by placing the piece of iron, whose specific gravity we are finding, on the pan; this will make the whole apparatus sink, so that the point o falls below the surface. If now we take off from the pan a sufficient weight, o may be made to rise to its position level with the surface. We will suppose the weight we have to remove from the pan is 200 grains. As this has been replaced by the iron this must be the weight of the iron in air.

Our third step is to find the weight of the iron in water or the weight of a volume of water equal to the volume of the iron. This is done by placing the iron on the cone at m.

The weight of the whole is not altered, but we find that the marked point o rises simply because, as we have already noticed, the iron appears to lose weight when weighed in water. To make up this apparent loss of weight and to bring o to its position level with the surface we must add weights. In this case the weight to be added is 27·77 grains. We have now—

1. Weight of iron in air = 200 grains.
2. Loss of weight in water = 27·77 grains.

Therefore in our equation—

$$\frac{W}{W - w} = S$$

we have—

$$W = 200$$

$$W - w = 27{\cdot}77$$

$$\frac{200}{27{\cdot}77} = S$$

$$7{\cdot}202 = S$$

Example.—The standard weight of a Nicholson's hydrometer is 1250 grains. A small substance is placed in the upper pan, and it is found that 530 grains are needed to sink the instrument to the standard point; but when the substance is put into the lower pan 620 grains are required. What is the specific gravity of the substance? [*S. & A.*, 1884.

The point o is level with the water when 1250 grains are in the pan.

The substance is able to sink the instrument to its proper depth if placed in the upper pan and 530 grains are added to it.

Therefore the weight of the substance in air

$$= 1250 \text{ grains} - 530 \text{ grains} = 720 \text{ grains}.$$

The substance is placed in the lower pan and 620 grains are added to it.

Therefore the weight of the substance in water

$= 1250$ grains $- 620$ grains $= 630$ grains.

Therefore its loss of weight in water

$= 720$ grains $- 630$ grains $= 90$ grains.

Therefore in the equation—

$$\frac{W}{W - w} = S$$

$$W = 720$$
$$W - w = 90$$
$$\therefore \tfrac{720}{90} = S$$
$$\therefore S = 8$$

i.e. the specific gravity of the substance is 8.

In cases where the substances to be dealt with are lighter than water, a movable cage of wire is affixed to the cone C, so that the substance is prevented from rising to the surface of the water as it otherwise would. With this cage we can proceed as in the ordinary way.

A sinker can also be used, as in the balance.

Specific Gravity of Liquids.—This can be found by means of the specific-gravity bottle.

This is a bottle so constructed that it shall hold when full a given weight of water at the standard temperature. The weight of the bottle thus filled can easily be compared with the weight of the same bottle when filled with the given liquid.

Thus, suppose the bottle itself to weigh W grains, and to be able to hold 100 grains of water; when filled with water the total weight is (100 + W) grains. If the weight of the bottle when filled with the liquid, say milk, is (103 + W) grains we have—

1. Volume of water filling bottle weighs 100 grains
2. " milk " " 103 "

$$\therefore \frac{\text{weight of milk}}{\text{weight of water}} = \text{specific gravity of milk}$$

$$i.e. \ \frac{103}{100} = S$$

$$\therefore \ S = 1{\cdot}03$$

The same instrument can also be used to find the specific gravity of a powder. As before, let the weight of the bottle filled with water be (100 + W) grains. Take a certain quantity

of the powder, say 10 grains of powdered porcelain. Place this powdered porcelain in the bottle and fill up with water and weigh. We shall find the weight to be (105·8 + W) grains. Therefore the weight of the powder and water in the bottle is 105·8 grains.

We now want the weight of the water displaced by the powder. This we obtain from the equation—

$$\begin{aligned}&(\text{weight of powder} + \text{water in bottle}) - \text{weight of bottle}\\ &\quad = \text{weight of water filling bottle} + \text{weight of powder}\\ &\quad - \text{weight of water displaced by powder}\end{aligned}$$

$\therefore$ 105·8 grains = (100 + 10) grains − weight of water displaced.

$\therefore$ weight of water displaced = (110 − 105·8) grains = 4·2 grains.

$$\therefore \text{ s.g. of powder} = \frac{\text{weight of powder}}{\text{weight of water displaced}} = \frac{10}{4\cdot2}$$

$\therefore$ specific gravity required equals 2·38.

A second instrument by which the specific gravity of a liquid may be found is the hydrometer.

This consists of a hollow graduated straight stem. It is hollow in order that it may float in the liquid. It is loaded at the bottom end to cause it to preserve its vertical positions when floating.

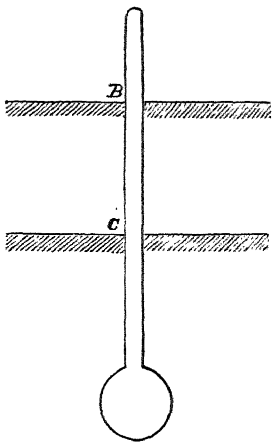


Fig. 90.

If it is successively made to float in different liquids, say water, milk, alcohol, etc., it will be found to sink to different depths, which can easily be determined by the graduations.

Let W be the weight of the instrument, A the area of a section of the stem.

If it sink to the mark B in water, then we know that W = weight of the volume of water displaced by part of instrument below B = V, say.

If it sink to the mark C in the liquid, then we know that W = weight of the volume of the liquid displaced by the part of the instrument below C = v, say.

But volume below C is equal to the volume below B – the part B C. Therefore $V = v$ – weight of volume of the liquid equal to the volume of B C.

Therefore if s be the specific gravity of the liquid, and the specific gravity of water is unity, we have—

$$V \times 1 = s(v - A \times BC)$$

$$\therefore s = \frac{V}{v - A \times BC}$$

A few special cases need attention, *e.g.* to determine the specific gravity of a substance affected by water.

The specific gravity of such a substance is found by finding its specific gravity relative to some liquid which does not affect it, the specific gravity of which is known, and then by multiplying the result thus found by the known specific gravity of the liquid used.

To illustrate this. We know that it would be useless for us to try to find the specific gravity of potassium by weighing it in water, as this element decomposes the water and forms with the water an entirely different compound.

To help us, we remember that potassium is unaffected by naphtha; we therefore find its specific gravity with respect to naphtha in the usual way, and then multiply the result found by the specific gravity of naphtha. The reason for so doing can be given thus—

Let W be the weight of a certain volume of potassium
" W_1 be the weight of the same volume of naphtha
" W_2 " " " " water

We want to find the value of the expression $\frac{W}{W_2}$

Our first step in the process gives us $\frac{W}{W_1}$, and the second gives us $\frac{W_1}{W_2}$

Therefore multiply together $\frac{W}{W_1} \times \frac{W_1}{W_2} = \frac{W}{W_2}$, and we get our needed result.

Example.—A piece of cupric sulphate weighs 3 ozs. *in vacuo* and 1·86 oz. in oil of turpentine. What is the specific gravity of cupric sulphate, that of oil of turpentine being 0·88?

[*Lond. Matric.*, 1868.

The sp. gr. of the cupric sulphate with respect to the oil of turpentine is found from the expression $\frac{W}{W - w} = S$, as in the ordinary way, *i.e.* sp. gr. of the cupric sulphate with respect to turpentine is $\frac{3}{3 - 1{\cdot}86} = \frac{3}{1{\cdot}14}$. This is the value in this case of the expression $\frac{W}{W_1}$

Again the expression $\frac{W_1}{W_2}$ is here equal to 0·88.

Therefore multiply $\frac{3}{1{\cdot}14}$ by 0·88 and we get the sp. gr. required, *i.e.* sp. gr. of cupric sulphate with respect to water

$$= \frac{3}{1{\cdot}14} \times 0{\cdot}88 = \frac{2{\cdot}64}{1{\cdot}14} = 2{\cdot}316$$

Specific Gravity of a Gas.—This can be done by means of a globe fitted with a stopcock in such a manner that the air can be exhausted by means of an air-pump. The operations needed are—

1. Exhaust the globe of its air by means of the air-pump and weigh. This gives the weight of the globe itself.

2. Fill the globe with the gas whose specific gravity we need and weigh again. We can now find the weight of the volume of the gas filling the globe.

3. Fill the globe with water and weigh.

We can now find the weight of the volume of water filling the globe, and we have all we need for the expression—

$$\frac{\text{Weight of globe full of gas}}{\text{Weight of globe full of water}}$$

which is the specific gravity required.

One fact should be remembered, though we have not space to enter into it fully.

To obtain accurate results the bodies should be weighed *in vacuo*, not in air; the air being a fluid, the weight of a body found by weighing it in air will be less than its real weight by the weight of air displaced.

Another useful problem in connection with specific gravity is the following—

A tube open at both ends is bent so that the two arms are parallel. It is placed with its open ends upwards, and liquids of different specific gravities are poured into it. Supposing

that the liquids will not mix, find the relative heights to which they will rise in the arms of the tube.

Let A be the point of junction of the two fluids; then if a horizontal line be drawn through A to B the line A B is in the same horizontal plane with B, and therefore the pressure at A must be equal to the pressure at B.

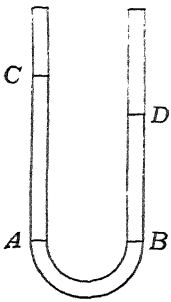


FIG. 91.

If h_1 be the length of A C
" h_2 " " B D

where C and D are the surfaces of the liquid.

The pressure at A = weight of the column A C

The pressure at B = weight of the column B D

seeing that the pressure of the atmosphere acts at both surfaces C and D, and therefore these forces neutralize each other.

But weight of column A C varies as $h_1 s_1$, where s_1 = sp. gr. of liquid in A C.

Weight of column B D varies as $h_2 s_2$, where s_2 = specific gravity of liquid in B D.

$$\therefore h_1 s_1 = h_2 s_2$$

Or—
$$h_1 : h_2 :: s_2 : s_1$$

$$\therefore h_1 : h_2 :: \frac{1}{s_1} : \frac{1}{s_2}$$

i.e. the heights of the columns are inversely proportional to the specific gravity of the liquids.

Example.—If the two liquids are water and oil (sp. gr. 0·88), and the water rises 11 inches above the common surface, to what height does the oil rise? [*S. & A.*, 1883.

Here we have to remember—

$$h_1 : h_2 :: \frac{1}{s_1} : \frac{1}{s_2}$$

But—
$$s_1 = 1$$
$$s_2 = 0·88$$
$$h_1 = 11$$

$$\therefore 11 : h_2 :: 1 : \frac{1}{0·88}$$

$$\therefore h_2 = \frac{11}{0·88} = 12\tfrac{1}{2}$$

Ans. $12\frac{1}{2}$ inches.

Examples on Specific Gravity.

Example 1.—A body weighs 60 grains in air, 48 grains in water, and 32 grains in another liquid. What is the specific gravity of the liquid?

Specific gravity of the body with respect to water is

$$\frac{60}{60-48} = \frac{60}{12} = 5$$

Also specific gravity of the body with respect to the liquid is

$$\frac{60}{60-32} = \frac{60}{28} = \frac{15}{7} = 2\tfrac{1}{7}$$

Therefore specific gravity of liquid is

$$\frac{5}{2\frac{1}{7}} = \frac{7}{3} = 2\tfrac{1}{3}$$

The reason for this last step will be seen from the following—

Let W be weight of volume of body = 60 grains;
let W_1 be weight of same volume of water = 12 grains;
let W_2 be weight of same volume of liquid = 28 grains.

We need the value of the expression $\frac{W_2}{W_1}$

From the facts given we can get the value of $\frac{W}{W_1}$, which equals 5; also the value of $\frac{W}{W_2}$, which equals $2\frac{1}{7}$.

But $\frac{W}{W_1} \div \frac{W}{W_2} = \frac{W_2}{W_1}$

Therefore we divide our first result by the second.

Example 2.—If a piece of wood weighing 120 lbs. floats in water, with four-fifths of its volume immersed, show what is its whole volume (1 cubic foot of water weighs 62·5 lbs.).

[*Lond. Matric.*, 1872.

We need to find here the specific gravity of the wood. This is at once found from the statement that it floats with four-fifths of its volume immersed, *i.e.* its weight is equal to the weight of four-fifths of the same volume of water.

i.e. if W be its weight, four-fifths of its volume of water = W.

$$\therefore \text{ its volume of water } = \frac{5W}{4}$$

$$\therefore \text{ its sp. gr.} = \frac{\text{weight of wood}}{\text{weight of same volume of water}} = \frac{W}{\frac{5W}{4}} = \frac{4}{5}$$

Therefore weight of 1 cubic foot of wood = $\frac{4}{5}$ weight of 1 cubic foot of water = $\frac{4}{5} \times 62{\cdot}5$ lbs. = 50 lbs.

Therefore number of cubic feet in the piece of wood = $\frac{120}{50}$ feet = $2\frac{2}{5}$ cubic feet.

Example 3.—The specific gravity of cork being 0·25, and that of brass 8; what is the weight of a piece of brass which when tied to a piece of cork weighing one pound will just sink it?

[*S. & A.*, 1893.

To answer this question fully, let us take as follows step by step.

(1) If sum of weights of the bodies = weight of whole combined

$$i.e.\ W_1 + W_2 = W$$

(2) If V_1, V_2, V = volumes

s_1, s_2, s = specific gravities

$$W = Vsw,\ W_1 = V_1 s_1 w,\ W_2 = V_2 s_2 w$$

$$\therefore\ V_1 s_1 w + V_2 s_2 w = (V_1 + V_2)\, sw$$

Where w = weight of cubic unit of standard, in our case, water.

$$\therefore\ 16 + V_2 \times 8 \times \tfrac{1000}{1728} = \left(\frac{16}{0{\cdot}25 \times \frac{1000}{1728}} + V_2\right) \times 1 \times \tfrac{1000}{1728}$$

for $s = 1$ = specific gravity of water

$$\therefore\ 16 - \frac{16}{0{\cdot}25} = V_2 \times \tfrac{1000}{1728}(-8 + 1) = -V_2 \times \tfrac{1000}{1728} \times 7$$

$$\therefore\ \frac{64 - 16}{7} = V_2 \times \tfrac{1000}{1728}$$

$$\tfrac{48}{7} \times \tfrac{1728}{1000} = V_2$$

$$\therefore\ W = V_2 s_2 w = \tfrac{48}{7} \times \tfrac{1728}{1000} \times 8 \times \tfrac{1000}{1728} \text{ ozs.}$$

$$= \tfrac{384}{7} \text{ ozs.} = 54\tfrac{6}{7} \text{ ozs.}$$

Ans. $54\frac{6}{7}$

Example 4.—A block of ice, volume of which is 1 cubic foot, is observed to float with one-twentieth of its volume above the surface, and a small stone (sp. gr. 2·5) is seen embedded in the ice; find the size of the stone, the specific gravity of ice being 0·9.

$$\text{Here } W_1 + W_2 = W$$

$$\text{also } V_1 s_1 w + V_2 s_2 w = Vsw$$

$$\text{or } V_1 s_1 + V_2 s_2 = Vs$$

$$\therefore\ (1728 \text{ inches} - V_2) \times 0{\cdot}9 + V_2 \times 2{\cdot}5 = 1728 \times \tfrac{19}{20} \text{ inches}$$

$$\therefore\ 1728 \text{ inches} \times \tfrac{9}{10} - V_2 \times \tfrac{9}{10} + V_2 \times \tfrac{5}{2} = 1728 \times \tfrac{19}{20} \text{ inches}$$

$$1728(\tfrac{9}{10} - \tfrac{19}{20}) \text{ inches} = V_2(\tfrac{9}{10} - \tfrac{5}{2})$$

$$1728\,(\tfrac{1}{20}) \text{ inches} = V_2\,(\tfrac{16}{10})$$

$$\therefore\ V_2 = \frac{1728 \times 10}{20 \times 16} \text{ inches} = \tfrac{1728}{32} \text{ inches}$$

$$\therefore\ V_2 = \tfrac{1}{32} \text{ cubic foot}$$

Example 5.—A man whose weight is 150 lbs. and specific gravity 1·1 just floats in water by help of a quantity of cork. The specific gravity of the cork is 0·24; find its volume.

Let V_1 = volume of cork and V_2 = volume of man in cubic feet

$$\text{then } V_1 s_1 + V_2 s_2 = (V_1 + V_2) s$$

$$\text{but } s = 1 = \text{specific gravity of water}$$

$$\therefore V_1 \times 0{\cdot}24 + V_2 \times 1{\cdot}1 = V_1 + V_2$$

$$\therefore V_2 \times (0{\cdot}1) = V_1 \times (0{\cdot}76)$$

$$\text{But } V_2 \times (1{\cdot}1) \times \tfrac{1000}{16} = 150$$

$$\therefore V_2 = \frac{\overset{3}{\cancel{150}} \times \overset{4}{\cancel{16}}}{1{\cdot}1 \times \underset{\underset{5}{\cancel{20}}}{\cancel{1000}}} = \frac{12}{5{\cdot}5} = \tfrac{24}{11}$$

$$\therefore \tfrac{24}{11} \times (0{\cdot}1) = V_1 \times (0{\cdot}76)$$

$$\therefore V_1 = \frac{24}{110 \times 0{\cdot}76} = \frac{6}{110 \times 0{\cdot}19} = \tfrac{60}{209}$$

Ans. $\frac{60}{209}$ cubic foot.

Examples on Specific Gravity.

(*a*) *Weight of Bodies.*

1. What do you understand by the specific gravity, or the specific density of a solid or liquid? [*S. & A.*

2. What is meant by saying that the specific gravity of oak is 0·7? What is the standard substance for specific gravity of solid and liquid—and for gases?

3. Explain the meaning of each letter in the formula W = V × specific gravity 1000 ÷ 16.

4. Find the weight of 192 cub. ins. of copper of which the specific gravity is 8·9. Also of 96 cub. ins. of mercury (specific gravity 13·5).

5. A rod of uniform cross-section 18 ins. long weighs 3 ozs., its specific gravity is 8·8; what fraction of a square inch is the area of the cross-section? (Cubic inch of water = 252 grs.) [*S. & A.*

6. Find the weight of a cube of iron, the edge of which is 3 ins., specific gravity of iron 8·8.

7. Given that a pint of water weighs 20 ozs., and that the specific gravity of proof spirit is 0·916; what fraction of a quart of proof spirit will weigh 30 ozs.? [*S. & A.*

8. The specific gravity of petroleum is 0·88 and a quart of water weighs 40 ozs. How many gallons of petroleum will weigh 38½ lbs.? [*S. & A.*

9. If 28 cub. ins. of water weigh 1 lb., what is the specific gravity of a substance of which 20 cub. ins. weigh 3 lbs.? [*S. & A.*

10. A casting is made of iron and weighs 4 cwt. When weighed in water it loses 80 lbs. Is it solid? If not, what is the united volume of the cavities within it? (Specific gravity of the iron 7·2.)

11. There are two spheres whose radii are as 1 to 2, and whose specific gravity as 7 to 2; compare their weights.

12. A flask holds 27 ozs. of water; what weight will it hold of an oil whose specific gravity is 0·95?

13. If a cubic inch of a standard substance weighs 0·75 of a lb., what is the weight of a cubic foot of a substance of which the specific gravity is 0·3?

(*b*) *Specific Gravity by weighing in Air and Water.*

14. Write out in words the formula for finding specific gravity by weighing in air and water. What is the name given to the principle referred to? Write out the statement.

15. A body weighs 16 lbs. in air and when immersed in water 13·25 lbs.; find its specific gravity.

16. A ball of platinum weighs 20·86 ozs. in air and 19·86 in water; find the specific gravity of the platinum.

17. A solid weighs 365 grs. in air, and 287 when immersed in water; what is its specific density?

18. A body weighing 450 grs. loses 210 grs. in water; what is its specific gravity?

19. The apparent weight of a piece of platinum in water is 60 grs., and the absolute weight of another piece twice as big as the former is 126 grs.; determine the specific gravity of platinum.

20. Explain briefly the method of finding the specific gravity of an insoluble body by means of the balance. If the body weighs 732 grs. *in vacuo* and 252 in water, what is its specific gravity? [*S. & A.*

(*c*) *Specific Gravity of Fluids.*

21. Describe the specific-gravity bottle, mentioning as many details as you can.

22. When the bottle is full of water, it is counterpoised by 983 grs. in addition to the counterpoise of the empty bottle, and by 773 grs. when filled with alcohol, what is the specific gravity of the alcohol? [*S. & A.*

23. A certain specific-gravity bottle weighs 1·37 gr., when full of water 187·63 grs., and when full of another liquid 142·71 grs.; what is the specific gravity of the liquid?

(*d*) *Specific Gravity of a Liquid by weighing Bodies in it.*

24. A piece of glass weighs 47 grs. in air, 22 grs. in water, and 25·8 in alcohol; find the specific gravity of the alcohol and state the general principles on which the solution depends.

25. If a ball of platinum weighs 20·8 ozs. in air, 19·86 in water, and 19·36 in sulphuric acid, what is the specific gravity of the acid?

26. A body of 3 lbs. weight *in vacuo*, and specific gravity 2·7 weighs 2 lbs. in a certain liquid; what is the specific gravity of the liquid?

27. A 6-lb. substance weighs 3 lbs. and 4 lbs. in two different fluids; compare the specific gravity of the fluids.

28. A glass ball weighs 69 grs. in pure water, 62 in sea water, and 130 in air; find the specific gravity of sea water.

29. A piece of glass rod weighs 35 grs. in air, and 21 in water; what is its weight in alcohol of specific gravity 0·9?

30. A body weighing 450 grs. *in vacuo*, loses 150 grs. in spirits, and 210 in water; find the specific gravity of the body and of the spirits.

31. A glass ball weighs 1000 grs., it weighs 630 grs. in water, and 650 in wine; find the specific gravity of the wine. [*S. & A.*

(*e*) *Specific Gravity of Bodies lighter than Water.*

32. Write out a formula, and explain each part, for working examples of this description.

33. The specific gravity of brass is 7·8. A piece weighing 2 lbs. is attached to a ball of wax weighing 3 ozs. The two when completely immersed in water together weighs 25·78; find the specific gravity of the wax.

34. A piece of mahogany weighing 375 grs., is tied to a piece of brass which weighs 380 grs. in water. The two together weigh 300 grs. in water; find the specific gravity of the mahogany.

35. A piece of wood weighing 12 lbs. is tied to a bit of lead which weighs 22 lbs. in air. The two together weigh 8 lbs. in water; find the specific gravity of the wood if the specific gravity of lead is 11·35.

36. A piece of iron, specific gravity 7·21, and weighing 360·5 grms., is tied to a piece of wood weighing 300 grms., and the weight of both in water is 110·5 grms.; find the specific gravity of the wood.

(*f*) *Bodies soluble in Water.*

37. How do you find the specific gravity of a body soluble in water? A body soluble in water weighs 29·5 grs. in air, and 28·5 grs. in spirits of specific gravity 0·86; what is the specific gravity of the substance?

38. The relative density of sodium to alcohol is 1·23, and of water to alcohol 1·27; find the specific gravity of sodium.

(*g*) *Specific Gravity of Mixtures.*

39. Liquids of specific gravity 1·2, 0·08, and 0·94 are mixed in the proportion of 3, 5, and 7; what is the specific gravity of the mixture?

40. The specific gravity of milk is 1·02. A quantity adulterated with water is found to have a specific gravity of 1·015; what proportion of water has been used?

41. The specific gravity of pure gold is 19·4, and of copper 8·8; what should be the specific gravity of a sovereign in which the proportion of gold to copper is 11 to 1?

42. The specific gravity of sulphuric acid is 1·84. Water and sulphuric acid are mixed in the proportion of 3 to 7, and the specific gravity of the mixture is found to be 1·6; how much has the mixture contracted?

43. A metal of specific gravity 15 is mixed with half its volume of another metal whose specific gravity is 12; find the specific gravity of the mixture.

44. Two substances whose specific gravity are 1·5 and 3 are mixed, and form a compound whose specific gravity is 2·5; compare the volumes and weights of the respective substances.

45. How much pure water must be added to sea water of specific gravity 1·027, that the specific gravity of the mixture may be reduced to 1·009?

46. If given quantities of two liquids of given densities are mixed, and

neither shrinking nor expansion takes place, how is the density of the mixture calculated? If 8 cub. ins. of a liquid (sp. gr. 1·25) are mixed with 12 cub. ins. of another liquid (sp. gr. 1·125), what is the specific gravity of the mixture if there be neither expansion nor contraction? [*S. & A.*

(*h*) *Hydrometers.*

47. Sketch a Nicholson's hydrometer; name all the parts and mention their use. How could you weigh ordinary small bodies with this instrument?

48. When loaded with 200 grs. in the upper pan, the instrument sinks to the marked point. A stone is placed in the upper pan, and the weight required to sink it to the same point is found to be 80 grs.; the stone is then placed in the lower pan, and the weight required is 128 grs.; find the specific gravity of the stone.

49. A Nicholson's hydrometer weighs 8 ozs. The addition of 2 ozs. to the upper pan causes it to sink in one liquid to the marked point, while 5 ozs. are required to produce the like result in another liquid; compare the specific gravities of the liquids.

50. In a Nicholson's hydrometer if the specific gravity of the weights is 8, what weight must be placed in the lower pan to produce the same effect as 2 ozs. in the upper pan?

51. In a Nicholson's hydrometer the standard weight is 1350 grs. A substance is placed in the upper pan, and 475 grs. have to be removed in order that the instrument may rise to the marked point. If now the substance be placed in the lower pan it is found there are 1200 grs. in the upper pan when the instrument returns to the marked point. Find the specific gravity of the substance.

52. The standard weight being 1200 grs., a body is placed in the upper pan, and it is found that 200 grs. must be added to sink the hydrometer to the standard point. The body is now placed in the lower pan, and it is found that 450 grs. must be placed in the upper pan to sink the instrument to the standard point. What is the specific gravity of the body? [*S. & A.*

53. The standard weight is 1250 grs., a small substance is placed in the upper pan, and it is found that 530 grs. are needed to sink the instrument to the standard point, but when the substance is put into the lower pan 620 grs. are required; what is the specific gravity of the substance? [*S. & A.*

54. Describe briefly an ordinary hydrometer of variable immersion, and state how it is used for finding the specific gravity of a liquid. Note the essential difference between such and a Nicholson's hydrometer. [*S. & A.*

(*i*) *Tensions in Strings, etc.*

55. A body weighing 12 lbs., and having a specific gravity 0·75, is fastened by a thread to the bottom of a vessel. When water is poured in so that the body is completely covered, to what tension is the thread exposed? [*S. & A.*

56. A body whose weight is 12 lbs., and specific gravity 2·5, is placed in a vessel with a horizontal base containing water; what pressure is exerted on the base by the body, supposing the body to be quite covered with water? [*S. & A.*

57. When a body is wholly or partially immersed in a liquid, what is the magnitude of the resultant pressure of the liquid upon it? A body whose specific gravity is 1·4, and volume 3 cub. ft. is placed in a vessel in which there is water enough to cover it; what pressure does the body produce on the points of the bottom at which it is supported? [*S. & A.*

58. A body whose volume is 5 cub. ft., and specific gravity 1·2, hangs by a string suspended in water which completely covers it; what force exerted along the string is needed to support the body? [*S. & A.*

59. A body weighing 18 lbs., specific gravity 3, is suspended by a string; what will be the tension in the string when it is suspended (*a*) in water, (*b*) in a liquid twice as dense as water?

(*k*) *Miscellaneous.*

60. A body weighs 2300 grs. in air, 1100 in water, and 1300 in spirit; what is the specific gravity of the spirit? [*S. & A.*

61. A glass ball weighs 3000 grs. and has a specific gravity of 3; what will be its apparent weight when immersed in a liquid whose specific gravity is 0·92? [*S. & A.*

62. If 5 cub. ins. of mercury weigh 2·45 lbs., and 2 cub. ins. of cast iron weigh 0·52 lb., what ratio does the density of mercury bear to that of cast iron? [*S. & A.*

63. A circular piece of gold and a common cork have equal weights and diameters, the cork is $1\frac{3}{4}$ in. long; how thick is the piece of gold (specific gravity of gold 19·25, of cork 0·23)? [*S. & A.*

64. Assuming that atmospheric air under ordinary conditions of temperature and pressure weighs 0·31 gr., and hydrogen 0·02 gr. per cub. in., what will be the weight of a bladder in which are 300 cub. ins. of hydrogen, that will just stay where it is placed, *e.g.* on the table? [*S. & A.*

65. The two parallel legs of a bent tube are held vertically, and liquids which do not mix are poured slowly into the tube, the heavier being poured first; find in what position they come to rest. If the two liquids are water and oil (sp. gr. 0·88), and the water rises 11 ins. above the common surface, to what height does the oil rise? [*S. & A.*

66. If 28 cub. ins. of water weigh 1 lb., what will be the specific gravity of a substance of which 20 cub. ins. weigh 3 lbs.? [*S. & A.*

67. A piece of silver and a piece of gold are suspended from the two ends of an equal armed balance beam which is in equilibrium when the silver is immersed in alcohol (sp. gr. 0·85), and the gold in nitric acid (sp. gr. 1·5); the densities of the silver and gold being 10·5 and 19·3 respectively, what are their relative masses?

68. A solid metal cylinder floats in mercury of specific gravity 13·6 with one-third of its length above the surface; find the specific gravity of the metal.

69. How much of its weight will a block of lead 30 cub. ins. in volume lose when weighed in alcohol (sp. gr. 0·8), the specific gravity of lead being 11·35?

70. A piece of lead of specific gravity 11·35 weighing 567·5 grs. is placed on the top of a floating block whose weight is 100 grs. so as to submerge both. The weight of both in water is 217·5 grs. Find the specific gravity of the block.

CHAPTER XVII.

CAPILLARY ATTRACTION.

TAKE a tube of very fine bore closed at one end and fill it with any liquid. Let us examine closely the surface of the liquid in the tube. We find it to be convex or concave instead of horizontal. This effect will be the more marked the narrower the bore of the tube.

This is one out of many instances of phenomena observed when solids are brought into contact with liquids. To the whole class of such phenomena is given the name of *capillary phenomena.* The word "capillary" is derived from the Latin word *capilla*, "a hair," and is used because these phenomena are best seen when the bodies used are tubes of extremely fine bore.

The phenomena can be divided into two sections—

1. When the liquid used *wets* the tube.

2. When the liquid used *does not wet* the tube, each of these sections can be subdivided into—

(1) The appearances presented by the surface of the liquid on the outside of the tube plunged into the liquid.

(2) The appearances presented by the surface of the liquid inside the tube.

To illustrate these effects take a series of tubes of different diameters open at both ends and use liquids: some, such as mercury, which will not wet the glass, others, such as water, which do wet the glass. We find the following :—

1. Place the glass tube in water; we notice a slight elevation on the outside of the glass where the glass is in contact with the water, and the surface of the water here, instead of being horizontal, is slightly concave.

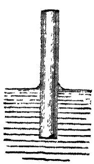


FIG. 92.

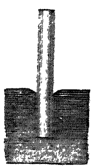


FIG. 93.

2. Place the same tube in a vessel containing mercury; we now notice just the opposite effect, *i.e.* a slight depression on the outside of the glass where the glass is in contact with the mercury, and the surface of the mercury here, instead of being horizontal, is slightly convex.

Again notice the effects inside the tube.

1. In the case of the water we find a considerable elevation above the outside surface of the water, and the finer the bore of the tube the higher the elevation.

2. In the case of the mercury we find the opposite effect, *i.e.* we find a considerable depression below the outside surface of the mercury, and the finer the bore of the tube the lower the depression.

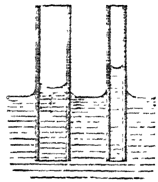


FIG. 94.

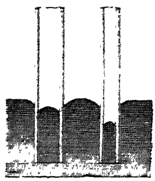


FIG. 95.

These effects have been found to obey certain laws—

1. The elevations and depressions caused are different as different liquids are used, water giving the highest elevation in cases where the liquids wet the glass.

2. Using the same liquid, the height of the elevation varies inversely as the diameter of the tube, and is not affected by the thickness of the glass or other material used.

To illustrate this. Suppose that in a tube $\frac{1}{20}$ of an inch in diameter the height reached is 1 inch; then in a tube $\frac{1}{40}$ inch in diameter the height reached will be 2 inches, and in a tube $\frac{1}{60}$ inch in diameter the height is 3 inches, and so on.

This law is called Jurin's law.

With regard to liquids which do not wet the glass Jurin's law is not found to be so correct, probably because there is a very thin layer of air between the tube and the liquid used, and this interferes with the law given.

Two or three theories have been given to show why the elevation or depression noticed takes place, and to explain the form assumed by the surface.

One is that the form of the surface depends upon the relation between the attraction of the solid for the liquid and the mutual attraction existing between the different particles of the liquid. Another is that these capillary phenomena are really cases of *surface tension.*

The subject of *surface tension* forms no part of our present work, and want of space forbids us entering upon it.

Capillary phenomena can be exhibited by plunging plates into different liquids, so as to be parallel to each other, and to be at different distances apart. Elevations and depressions will be noticed, varying inversely as the distance which separates them.

Instances of capillary attraction are found in the manner in which the oil rises in the wick of the lamp, in the way in which the water appears to be sucked up into the sponge, or the sauce on the plate as it rises into the salt, and so in many other cases which will occur at once to the mind of the student.

CHAPTER XVIII

PNEUMATICS.

WE have now to deal with the other class of fluids, viz. gases. These possess the properties—weight, elasticity, compressibility, etc., as can be easily shown by experiment.

The weight of a gas can be shown as follows:—

Take a glass vessel which can be fitted with a stopcock; weigh it; exhaust the air by means of the air-pump and weigh again. In this case we find a decrease in weight, which can only be accounted for by the removal of the air present at the first weighing. The pressure of the atmosphere is another instance of the weight of a gas; illustrations of the use of this pressure are furnished by the ordinary pump, the barometer, the siphon, etc., which will be discussed in due course.

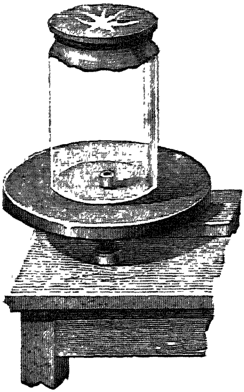


FIG. 96.

It has been found by experiment that the atmosphere presses on every object with a force equal to 15 pounds on each square inch of surface; in other words, if the column of air standing on 1 square inch be weighed it will be found to weigh 15 pounds. The method by which it can be weighed will be explained when we come to the barometer.

To show the great force which the atmosphere exerts take a short cylinder of glass open at both ends. Let one end be closed so as to be air-tight by means of a piece of tissue-paper or bladder. We have now

two forces acting on the covering, one on each side; these two, being the pressure of the atmosphere, are equal, but in opposite directions, one on the inside, the other on the outside. Let this inside force be removed by exhausting the air by means of the air-pump. During the experiment we shall be able to work the air-pump until at length we reach the point when the outside pressure of the atmosphere, *i.e.* 15 pounds on the square inch, becomes too great for the covering to sustain. The covering bursts with very great violence, and the forces are again equalized.

Another very simple experiment illustrates the same thing. Take an ordinary tumbler; fill it completely with water. Cover it, as it stands, so as to be air-tight with a piece of writing-paper. If the experiment be carefully done, the tumbler can be turned upside down, and yet the water be kept from issuing from the tumbler. Why is this? The paper is acted upon by two unequal forces in opposite directions—

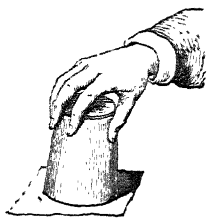


FIG. 97.

1. The weight of the water in the tumbler, which is the less force, acting downwards so as to force away the paper and so obtain an exit.

2. The pressure of the atmosphere, which is the greater force, being equal to 15 pounds on the square inch, acting upwards so as to force the paper towards the tumbler, and so prevent the exit of the water.

This state will continue whilst the paper forms an air-tight covering for the tumbler.

Another very instructive example is furnished by the Magdeburg hemispheres. These consist of two hollow brass hemispheres fitting the one into the other. One is furnished with a stopcock, by means of which it can be attached to the air-pump, and so the two can be exhausted of the air contained therein.

We notice two very different states of the hemispheres—

1. Before the air-pump is worked, the hemispheres can be very easily separated.

2. After the air has been exhausted, they cannot be separated except by a very powerful force.

This experiment can be explained in a manner similar to that which has already been given.

We have already spoken of the elasticity and compressibility of the air. These properties can be beautifully illustrated

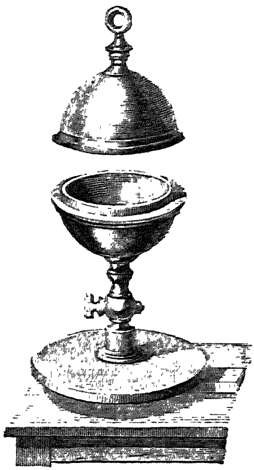


FIG. 98.

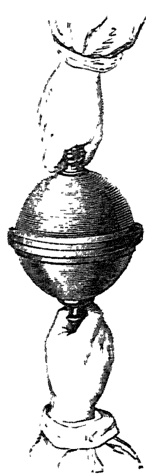


FIG. 99.

by placing a bladder partially filled with air under the receiver of an air-pump. Having placed the bladder in its proper position, we notice that before commencing to exhaust the air from the receiver the pressures on the bladder from the inside and the outside are equal. As we exhaust the air from the receiver, the quantity of air outside the bladder diminishes, and therefore becomes rarer, because it expands, and the pressure on the bladder from the outside decreases, and therefore the air within the bladder expands, and the bladder itself is blown out. Allow the air to fill the receiver again; at once the bladder collapses and regains its original size and shape.

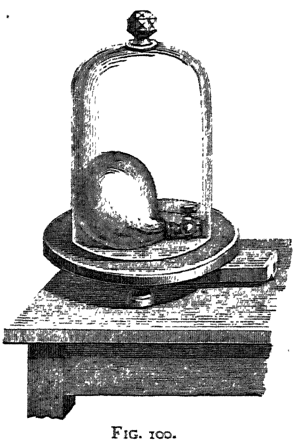


FIG. 100.

When speaking of matter possessing the property of compressibility, we used the illustration of a cylinder made from a stout piece of glass tubing fitted with a piston. When the pressure is removed from the piston, the piston at once returns to its original position, showing the elasticity of the air in regaining its original volume. To illustrate the elasticity of gases more fully we will use the following, which is generally known as Boyle's or Mariotte's law:—

If the temperature of a gas be kept the same, its volume is inversely proportional to the pressure, *e.g.* 1 cubic foot of air when subjected to the pressure of one atmosphere, *i.e.* a pressure equal to 15 pounds on the square inch, occupies half a cubic foot when subjected to the pressure of two atmospheres, *i.e.* to a pressure of 30 pounds on the square inch, and so on; or generally—

If V be the volume at pressure P, and v that at pressure p, then—

$$V : v :: p : P$$

or,

$$V : v :: \frac{1}{P} : \frac{1}{p}$$

or,

$$VP = vp$$

This can easily be remembered by noticing that if we multiply together the volume and the pressure producing the volume we get the same result equal in every case when speaking of the same gas at the same temperature. Taking the cylinder made from the glass tubing again; suppose the length of the tube between the piston and the closed end to be 6 feet when the pressure on the piston is 1 lb.; if we double the pressure this length will be reduced to 3 feet; if we make the pressure 3 lbs., then this length will be reduced to 2 feet, etc.

The law stated can be verified by the following experiment. Take a piece of glass tubing of uniform bore, closed at one end and open at the other. Bend it twice at right angles, so that the leg A B, which is closed, may be shorter than the open leg C D. Suppose the barometer to indicate 30 inches of mercury, *i.e.* as we shall shortly show, the weight of a column of mercury 1 square inch in section and 30 inches long, is equal to the pressure of the atmosphere on each square inch of surface exposed. Now, into our prepared tube pour a quantity of mercury until it stands at the same height in each leg, *i.e.* it now stands at the mark *a b*. As *a b* is in the same horizontal plane, the surface *a* must be acted upon by the same

pressure as the surface b, viz. the pressure of the atmosphere equal to the column of mercury 30 inches long, *i.e.* the air within B b is subject to this pressure. Now pour into the tube more mercury. We shall find that the two columns in A B and C D do not rise at the same rate: when the surface of the one in A B is at e, the one at C D is at d. The reason for this is that the two surfaces are now subjected to different pressures: that at d, being exposed to the air, is subject to the pressure of the atmosphere only; but the one at C is subject to this pressure, and another in addition, viz. the pressure due to the weight of the column of mercury, the height of which is dc, where e is in the same horizontal plane with c. Suppose the length of dc is l.

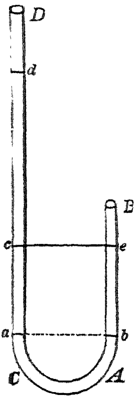


FIG. 101.

Then if the law stated be true, the following proportion should be true—

Volume of air in Bb : volume of air in Be : : $30 + l : 30$;
or, as the tube is of uniform bore, we may write this—

$$Bb : Be :: 30 + l : 30$$

This is found to be the case.

If we make the proportion given general by writing h for 30, our result will still be true, and will serve for any state of the atmosphere; we then have—

$$Bb : Be :: h + l : h$$

Example.—The barometer stands at 29 inches. Supposing B b to be 6 inches long, and more mercury to be poured into the tube until the mercury has risen to e, so that B e is 4 inches. Find the length of $d\,c$, and give the pressure upon the air in B e.

Here we have—

$$6 : 4 :: 29 + l : 29$$

from which

$$3 : 2 :: 29 + l : 29$$
$$87 = 58 + 2l$$
$$29 = 2l$$
$$l = 14\tfrac{1}{2}$$

The length of $d\,c$ is therefore $14\frac{1}{2}$ inches.

The air in B b is subject to a pressure equal to the weight of a column of mercury in height $(29 + 14\frac{1}{2})$ inches, *i.e.* $43\frac{1}{2}$ inches.

Boyle's law is true for air and some other gases, except where the pressures exerted are very large, then small devi-

ations from the law are noticed. There are certain gases, such as carbonic acid gas, which become liquid at moderate pressures; here the law is subject to serious modifications, especially near the point where liquefaction commences.

The Diving-bell.—Take an ordinary tumbler and hold it vertically, mouth downwards. If we attempt to press it into water whilst held in this manner, we shall find resistance is offered, owing to the elasticity of the air in the tumbler. As the force we use is increased, the volume of the air in the tumbler is proportionally decreased. This simple illustration of Boyle's law is really an explanation of the action of the diving-bell. As the tumbler descends, the volume of air in it diminishes, and therefore it becomes partly filled with water. To obviate this the diving-bell is provided with apparatus by means of which air is forced into the bell from the surface of the water. Air is thus forced in until the water is brought nearly to the mouth of the bell, and the bell becomes filled with air in a compressed state.

Example.—What is a water barometer? A cylinder, 11 feet long, and open at one end, is held with its open end downward in water, and is just immersed; the water is kept from getting into the cylinder by forcing in more air; the water barometer stands at 33 feet. Compare the quantity of air forced in with the quantity originally in the cylinder. [*S. & A.*, 1893.

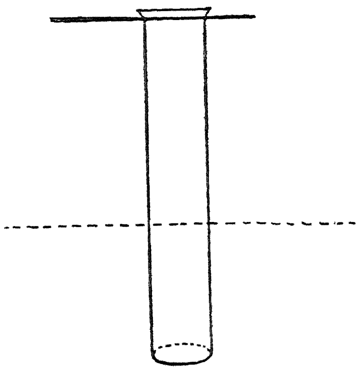


FIG. 102.

Let V_1 = volume of cylinder, then by Boyle's law—

$$V_1P_1 = V_2P_2$$

where P_1, P_2 are pressures under first and second conditions respectively, and V_2 is volume of air under second condition, *i.e.* when the cylinder is just immersed.

Now P_1 is measured by water barometer and is found to be 33 feet;

also P_2 is measured by $(33 + 11)$ feet = 44 feet

$$\therefore V_1 \times 33 = V_2 \times 44$$
$$\therefore V_2 = V_1 \times \tfrac{3}{4}$$

Therefore under the extra pressure, the air would be compressed to three-quarters its original volume. The air that must be forced into it to prevent water from getting in must be one quarter original volume (at increased pressure); but this volume at increased pressure

$$= \tfrac{1}{4}V_1 \times \tfrac{4}{3} \text{ at atmospheric pressure}$$
$$\therefore \text{answer required} = \tfrac{1}{3}V_1$$

Examples on Chapter XVII.

Weight of Air.

1. Describe some simple experiments to show that air has weight. Was it always believed that this was the case? How do you account for the mistake?

2. What is the weight of a cubic foot of air? What is the specific gravity of air compared with pure water?

3. What do you mean by an elastic fluid. Is steam an elastic fluid?

Boyle's Law.

4. What is meant by the expression, "VP is constant"? Describe Boyle's or Mariotte's tube for the experimental verification of this law.

5. Is this law true under all circumstances? For instance, for very great pressures? What do you understand by the "critical point"?

6. For what gases is this law very approximately true? For which gases is it not so true?

7. Suppose a vessel contained 2 pints of oxygen, and that 2 pints of hydrogen are also introduced; what pressure would the hydrogen exert on the sides of the vessel, considering that the 2 pints of oxygen are already present?

8. Is this law applicable when the temperature is varying? Write out the statement for this law as clearly as possible.

9. State the relation between the pressure and volume of a gas at a given temperature. Is this law exactly true? Give reasons for your answer. [*S. & A.*

10. A cylinder is fitted with a piston, which works in it easily and air-tight; when the water barometer is at 33 ft., the piston is 1 ft. above the bottom of the cylinder. If the cylinder were slowly sunk in water to a depth of 44 ft., how high would the piston be above the bottom of the cylinder? [*S. & A.*

11. A tube filled with water is inverted with its open end in water, no air having got in; the top of the tube is 20 ft. above the surface of the external water. If the water barometer stands at 34 ft., what is the pressure in pounds per square foot at a point inside of the top of the tube? [*S. & A.*

12. What would be the consequence of making a hole through the top of the tube in previous question; and why should the consequence follow? [*S. & A.*

13. A hollow cylinder is full of air at a pressure of 15 lbs. to the square inch when the piston is 12 ins. from the bottom; if more air is

forced in till there is three times as much air as at first, and if the piston is allowed to rise 4 ins., what is now the pressure of the air per square inch? [*S. & A.*

14. A barometer stands at 30 ins.; the vacuum above the mercury being perfect; the area of the cross-section of the tube is one quarter of a square inch; if one quarter of a cubic inch of the external air is allowed to get into the barometer, and the mercury is found to fall 4 ins., what was the volume of the original vacuum? [*S. & A.*

15. In a bent tube, open at one end and closed at the other, the mercury stands at the same level in both branches, and the contained air occupies 30 cms. at the ordinary pressure of 76 cms. If the section of the tube be 10 sq. cms., what volume of mercury must be poured into the tube to compress the air 20 cms.?

16. The mercury in a barometer stands at 30 ins., and the sectional area of the tube is 1 sq. in. A cubic inch of air is admitted through the mercury into the vacuum space, and depresses the column 4 ins. Find the size of the vacuum.

17. A quantity of air has a volume of 12 cub. ins. under a pressure of 15 lbs. to the square inch. What will be the volume under a pressure of 25 lbs.?

18. A cylindrical tube 4 ft. long is closed at one end, and when full of air is closed at the other end by a tightly fitting piston, having an area of 3 sq. ins., and weighing 9 lbs. How far will the piston descend by its own weight—the cylinder being vertical? If a force of 24 lbs. be applied to the piston, how far will it then descend?

19. The area of the cross section of a barometer tube is one-fifth of a square inch, and the vacuum above the column 10 ins. long. How far will the mercury sink if 2 cub. ins. of the external air be admitted into the vacuum? Barometer stands at 30 ins.

20. In the above question what volume of atmospheric air must be introduced to cause the mercurial column to fall 4 ins.

21. The area of the cross-section of a barometer tube is one-third of a square inch, and the vacuum at the top is perfect. One cubic inch of atmospheric air is introduced, and the column falls 9 ins. Find the lengths of the original vacuum. Barometer stands at 30.

Diving Bells.

22. A cylindrical diving-bell is lowered into the sea until the level of the water within it is 85 ft. below the surface of the sea. Find the pressure within the bell, it being given that the pressure of the atmosphere = 15 lbs. per square inch = 32 ft. of sea water.

23. To what depth must the top of a diving-bell 8 ft. high be immersed under water that the air may be compressed to half its volume—the height of water barometer being 34 ft.

24. A cylindrical bell 6 ft. high is furnished with a barometer standing at 30 ins. If the bell be lowered into water till the barometer stands at 40 ins., find the depth of the top of the bell below the surface of the water (specific gravity of mercury 13·6).

25. What additional volume of air at the ordinary pressure (34 ft.) must be admitted into a bell 8 ft. high, the internal section of which is uniform and equal to 20 square feet, and the top 60 ft below the surface. to completely fill it?

CHAPTER XIX.

HEAT—TEMPERATURE—THERMOMETERS, ETC.

In our last chapter we have spoken of the volume of a gas being inversely proportional to the pressure acting upon it, *if the temperature of the gas be kept the same.* This is a very important condition, and one that needs consideration.

Let us then first explain the term *temperature*, and in so doing we must be careful not to confuse it with the term *heat.*

I touch the inkstand which I am using, the paper on which I write, the desk at which I sit; I find a series of different sensations produced—the inkstand is cold, the paper not quite so cold, the desk slightly warmer than the paper. Place the same articles near the fire, and again touch as before, the sensations are different altogether. What has caused the difference? A difference of condition of the articles has been produced. What has caused the difference of condition?

The *agent* which has produced the change of condition is called *heat.* The condition of the body as regards its heat is called its *temperature.*

Thus *heat* is the name given to an *agent*, a *force; temperature* is the name given to the *condition* of the body as regards the heat.

We have spoken of "warmer" and "colder" bodies. The desk was said to be slightly warmer than the paper, the inkstand slightly colder than the paper. The inkstand brought near the fire becomes warmer.

When two bodies are brought into contact, one being warmer than the other, we find that after a time they become as warm as each other; the temperature of one is lowered, whilst that of the other is raised. A passage of heat from one body to the other has taken place. The warmer body has given of its heat to the other, the colder body. When one body is warmer than another in contact with it, and loses part of its heat to the colder body, we say that it had a *higher temperature*, or that the one which was colder, and has received heat, had a *lower temperature.*

We have replaced the terms *warmer* and *colder* for *higher* and *lower temperature* respectively. If the two bodies, being in contact, and no heat passes from the one to the other, we say that their *temperatures are equal.*

From what has been said, the distinction between heat and temperature should be clear.

The relative temperatures of the different bodies have been measured by the hand, *i.e.* the sensations produced by touching the different bodies gave a rough idea of the relative conditions with regard to heat. We have thus used the hand as a *measurer* of temperature, but, as will readily be seen, the hand would prove a very poor instrument to be used generally. We need an instrument which will be reliable, and which will give us not a rough idea, but a definite statement of the facts we wish to know. How shall we obtain such an instrument?

Notice the rails on the line of railway. We see that their ends are not close together. Why is this? Because the heat

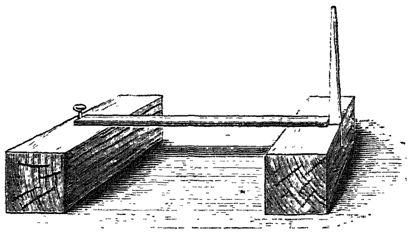


FIG. 103.

of the sun will lengthen them to a certain extent. This lengthening must be allowed for. Take a thin rod of iron, place it as in the figure, with one end firmly fixed, the other

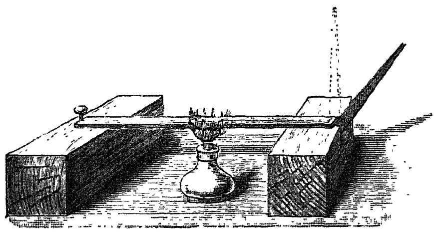


FIG. 104.

end not fixed. Place a spirit lamp beneath the rod, between the blocks. Now, with a proper method of measuring the different lengths of the rod, we shall find that as the rod

becomes hotter it gets longer; as it cools it returns to its original length.

These two experiments give us a principle by which we can construct the needed instrument. This instrument is called a thermometer. The principle of its construction depends upon the fact that heat expands solids, liquids, and gases.

Heat expands solids, as we have illustrated by the bar of iron. It expands liquids, as will be seen in the "mercurial thermometer." It expands gases, therefore the need for the insertion of the sentence at the beginning of the chapter.

We need not discuss the different materials of which a thermometer might be made; the bar of iron mentioned would not do, because of the small elongation for the amount of heat used. Water might be used up to a certain heat, but we know that at a certain point it boils and passes into vapour; the same remark applies to alcohol. Mercury is found to be the most suitable liquid for ordinary purposes.

There are certain conditions which must be attended to in the making of the mercurial thermometer.

1. The tube of the thermometer should have a small bulb and a very fine bore. A large flask would be inconveniently heavy, and the mercury would take a long time to heat; the fineness of the bore will help us to a rise in the tube, clearly manifest on a small increase of temperature.

2. The air must be removed from the tube, as any air not removed would tend to prevent the rise of the liquid, and thus wrong measurement would be given.

3. The tube must be closed, as the mercury must be protected from the pressure and influence of the atmosphere. The pressure of the atmosphere would of itself change the length of the thread of mercury; besides which, impurities would get into the liquid.

We have now to explain the making or graduation of the instrument. This must be done in order to show the elongation of the thread of liquid corresponding to a change of temperature. As the instrument is to be of universal use, and to be a standard by which all different persons may work, there must be certain fixed points of temperature agreed upon, from which the lengths of the thread may be measured and the different elongations noted.

These are obtained from the physical facts connected with (1) melting ice, and (2) steam of boiling point.

The temperature of melting ice is the same at all times and in all places; the temperature of the steam of boiling water is the same at all times and in all places if the pressure on the water is the same, *i.e.* the pressure of the atmosphere being the same, water will always boil at one fixed temperature, and allowance must be made for any variation.

Special care must be taken to remember that at different pressures we should obtain different fixed points, and some *standard* pressure must be chosen, at which pressure the fixed points shall be determined.

In England the standard pressure is taken to be the pressure of 30 inches of mercury on the square inch. In other words, the barometer must stand at 30 inches, allowance being made for any variation.

These two fixed points are called—

1. The freezing point.
2. The boiling point.

The length of thread between the two can now be divided into equal parts, called degrees.

We have to consider three methods of division, or thermometric scales—

1. The Fahrenheit scale, the one in common use in England.
2. The Centigrade scale, the one in common use on the Continent and amongst scientific men in England.
3. Réaumur's scale, the one in common use in Germany.

On the Fahrenheit (F.) scale, the freezing point is 32°, and the boiling point 212°, giving an interval of 180° between the two fixed points.

On the Centigrade (C.) scale, the freezing point is 0°, and the boiling point, 100°, giving an interval of 100° between the two fixed points. All temperatures below the freezing point in this case will be negative, and are indicated thus—

$$-1^\circ, -8^\circ, -75^\circ, \text{etc.}$$

On the Réaumur (R.) scale the freezing point is 0°, and the boiling point 80°, giving an interval of 80° between the two fixed points. All temperatures below the freezing point in this case, as in the Centigrade, will be negative, and are indicated thus:

$$-5^\circ, -7^\circ, -60^\circ, \text{etc.}$$

Thus, according to the intervals given, we have—

$$180^\circ \text{ F.} = 100^\circ \text{ C.} = 80^\circ \text{ R.}$$
$$\text{or} \quad 18^\circ \text{ F.} = 10^\circ \text{ C.} = 8^\circ \text{ R.}$$
$$\text{or} \quad 9^\circ \text{ F.} = 5^\circ \text{ C.} = 4^\circ \text{ R.}$$

These figures will help us to change the notation, as read by one scale, to the reading of another, bearing in mind the difference of the figure at the freezing point, *i.e.*—

Freezing point F. = 32°
" " C. = 0°
" " R. = 0°

Thus we have—

$$\frac{\text{F.} - 32}{180} = \frac{\text{C.}}{100} = \frac{\text{R.}}{80}$$

where F., C., R., is the number of degrees according to each scale.

We can best illustrate this by giving a few examples.

Example 1.—A thermometer reads 75° F.; what would it read on the Centigrade scale? We have—

$$\frac{\text{F.} - 32}{180} = \frac{\text{C.}}{100}$$
$$\text{or } \frac{\text{F.} - 32}{9} = \frac{\text{C.}}{5}$$
$$\therefore \frac{75 - 32}{9} = \frac{\text{C.}}{5}$$
$$\therefore \text{C.} = \tfrac{5}{9}(75 - 32) = \tfrac{5}{9} \times 43 = \tfrac{215}{9} = 23 \cdot \dot{8}.$$

Example 2.—Convert 16° R. into Fahrenheit. Here—

$$\frac{\text{F.} - 32}{180} = \frac{\text{R.}}{80}$$
$$\text{or } \frac{\text{F.} - 32}{9} = \frac{\text{R.}}{4}$$
$$i.e. \ \frac{\text{F.} - 32}{9} = \frac{16}{4}$$
$$\frac{\text{F.} - 32}{9} = 4$$
$$\text{F.} - 32 = 36$$
$$\therefore \text{F.} = 68$$

Example 3.—What does -10 F. read on the Centigrade scale?

$$\frac{\text{F.} - 32}{9} = \frac{\text{C.}}{5}$$
$$\frac{-10 - 32}{9} = \frac{\text{C.}}{5}$$
$$\therefore \text{C.} = \tfrac{5}{9}(-10 - 32)$$
$$= -\tfrac{5}{9} \times 42$$
$$= -\tfrac{210}{9}$$
$$= -23\cdot\dot{3}.$$

Example 4.—At what temperature is the number on the Centigrade and Fahrenheit thermometers the same?

Let x be the degree marking the temperature. Then we have as before—

$$\frac{\text{F.} - 32}{9} = \frac{\text{C.}}{5}$$
$$\text{in this case } \frac{x - 32}{9} = \frac{x}{5}$$
$$\text{or } 5x - 160 = 9x$$
$$-160 = 4x$$
$$\therefore x = -40^\circ$$

Example 5.—At what temperature is the number on the Fahrenheit and Réaumur scales the same?

Let x = degree marking the temperature. Then, as before, we have—

$$\frac{\text{F.} - 32}{9} = \frac{\text{R.}}{4}$$
$$\text{or } \frac{x - 32}{9} = \frac{x}{4}$$
$$4x - 128 = 9x$$
$$-128 = 5x$$
$$\therefore x = -25\cdot6^\circ$$

TABLE TO HELP THE MEMORY.

	F.	C.	R.
Freezing point	32°	0°	0°
Boiling point	212°	100°	80°
Interval between fixed points	180°	100°	80°
Fraction showing readings on different scales	$\frac{F - 32}{180}$	$\frac{C}{100}$	$\frac{R}{80}$

Example 5.—Explain the graduation of the thermometer.
A temperature is 10° on Fahrenheit's scale; what is it on the Centigrade scale? [*S. & A.*, 1893.

To work out the example. Remember—

$$\frac{F. - 32}{180} = \frac{C.}{100}$$

$$\text{or } \frac{F. - 32}{9} = \frac{C.}{5}$$

$$\therefore \frac{10 - 32}{9} = \frac{C.}{5}$$

$$\text{or } 5 \times (10 - 32) = 9\,C.$$

$$- 5 \times 22 = 9\,C.$$

$$- \tfrac{110}{9} = C.$$

$$\text{or } C. = - 12\tfrac{2}{9}$$

Examples on Chapter XIX.

1. What do you mean by the "temperature" of a body? Is the "temperature" of a body another word for the heat of a body?
2. What is the relation of temperature to heat? Can you mention a body possessing no temperature?
3. When have two bodies the same temperature? Have they the same or equal temperatures when they possess the same quantity of heat?
4. Two bodies, A and B, are hot—but not at the same temperature. What is meant by saying that A has a higher temperature than B?
5. Does an iceberg "give off cold?" How is it that one feels so very chilly when close to an iceberg?
6. What is required to enable a body to pass from one condition to a higher, *e.g.* from solid to liquid, or liquid to gaseous?
7. A pound of ice at 0° C. is "warmed" over a fire until it is a pound of water at 0° C. Has it taken up any heat? What has become of it?
8. Write out, as near as you can, a definition of "equal temperatures."
9. Describe a thermometer, and say what it is used for. Does it measure the temperature of a room?
10. Why is mercury used commonly in thermometers? How do you find the freezing point and the boiling point?
11. What do you mean by Fahrenheit, Centigrade, and Réaumur's scales? Where are these used?
12. Where is the "freezing point" on each of these instruments, and where the boiling point?
13. Are these points the same under all circumstances?—for instance, does water always boil at 100° C.?
14. State distinctly how to convert Fahrenheit degrees to Centigrade degrees, and *vice versâ.*
15. Establish a connection between Fahrenheit, Centigrade, and Réaumur's scales, and express in Centigrade degrees: 80° R., 12° R., 20° F., 212° F., and 235° F.

16. What does $-13°$ F. mean? Express this in Centigrade. Also express in Centigrade: $-26°$ F., $-12°$ R., and $-40°$ R.
17. Change the following figures into Fahrenheit degrees: $14°$ R., $-12°$ R., $32°$ C., and $180°$ C.
18. Change into Réaumur: $250°$ C., $-10°$ C., $-40°$ C., and $-40°$ F.
19. Show that $-40°$ F. and $-40°$ C. indicate the same temperature.
20. When is the temperature of one body said to be higher than that of another, and when are they said to have equal temperatures?
21. What is the boiling point of a thermometer, and in what sense can it be called a fixed point? [*S. & A.*

CHAPTER XX.*

EXPANSION OF GASES.

WE have spoken of the effect of heat on solids and liquids. We have now to consider more fully its effect on gases. What is the effect? To answer this question, take a glass flask fitted with a cork; in the cork insert a long narrow glass tube, bent as in the figure. We wish to obtain a short length of coloured water in the tube. To do this, invert the flask with the hand, holding the tube whilst in the flask over a disc containing the coloured water. Take the hand for a short time from the flask. The coloured water rises in the tube. Take the flask in the hand again and set it on the table in its proper position, we now see a short length of coloured water in the tube as desired. This short strip of coloured water is to tell us the effect of heat on the air within the tube.

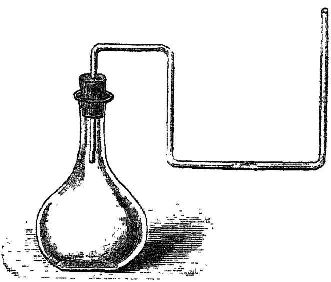


FIG. 105.

Take the flask in the warm hand. The coloured water moves away from the flask, telling us that the space occupied by the air in the flask has been increased, in other words, the warmth of the hand has expanded the air in the flask.

Place the flask in cold water, the coloured water moves towards the flask, showing that the air in the flask contracts

when cooled. We have constructed a simple piece of apparatus which may be used as a thermometer. This *air thermometer* can be made of very great service when small variations of heat are to be noticed.

We have shown that—

1. Air expands when heated.
2. Air contracts when cooled. We can take these statements as true of gases in general.

The laws which hold with regard to the expansion of gases are as follows:—

i. All gases have the same coefficient of expansion as air.

ii. The coefficient of expansion is the same whatever be the pressure.

They were discovered by Dalton, and independently by Gay-Lussac, to whom they are usually ascribed.

We have given a general statement of the effect of heat upon gases, *i.e.* expansion. What proportion does the amount of expansion bear to the original volume for a given increase of temperature? It has been found by experiment that the following is true using the Centigrade scale—

1 cubic in. of gas at 0° becomes $(1 + \frac{1}{273})$ cubic in. at 1°;
1 " " 0° " $(1 + \frac{2}{273})$ " 2°;
1 " " 0° " $(1 + \frac{3}{273})$ " 3°;

and generally—

1 cubic in. of gas at 0° becomes $\left(1 + \frac{x}{273}\right)$ cubic in. at x°.

This is true for all general purposes if the temperature does not much exceed 100° C. To express this in a verbal statement we have as follows—

The volume of a dry gas, whatever the pressure may be, is increased $\frac{1}{273}$ of its volume for every increase of 1° C. of temperature, always reckoning from 0° C.

This statement is called the law of Charles, who was really the first discoverer of (i.); or expressing it in a formula, we have volume V of a gas at 0° C. becomes $V\left(1 + \frac{t}{273}\right)$ at t° C., the pressure remaining the same.

EXAMPLES.

Example 1.—A gas measures 120 cubic inches at 0° C., what will it measure at 20° C., the pressure remaining constant?

$$\text{Volume at } 0^\circ = 120 \text{ cubic inches} = V.$$
$$\therefore \text{by formula, volume required} = V\left(1 + \frac{t}{273}\right)$$
$$\text{or } 120 \times (1 + \tfrac{20}{273}) \text{ cubic inches}$$
$$= \left(120 + \frac{20 \times 120}{273}\right) \text{ cubic inches}$$
$$= 120 + 8{\cdot}7 \text{ cubic inches}$$
$$= 128{\cdot}7 \text{ cubic inches.}$$

Example 2.—A gas measures 170 cubic inches at 0° C., what will it measure at 17° C., the pressure continuing the same?

$$\text{Volume at } 0^\circ = 170 \text{ cubic inches}$$
$$\therefore \text{Volume at } 17^\circ = (1 + \tfrac{17}{273}) \times V$$
$$\text{Volume at } 17^\circ = (1 + \tfrac{17}{273}) \times 170 \text{ cubic inches}$$
$$= (170 + 10{\cdot}5) \text{ cubic inches}$$
$$= 180{\cdot}5 \text{ cubic inches.}$$

We have considered the effect of pressure on gases, and that of heat on gases separately; we must now consider the effect of the two combined. Let us first recapitulate the results obtained.

1. *For pressure.*—If the temperature of a gas be kept the same, its volume is inversely proportional to the pressure (Boyle's law).

If V_1 be volume at pressure P
and V_2 ,, ,, p
then $V_1 : V_2 :: p : P$
or $V_1P = V_2p$ ....(1)

2. *For expansion by heat.*—If the same pressure be maintained, a gas increases $\frac{1}{273}$ of its volume for every increase of 1° C. of temperature, reckoning from 0° C. (Law of Charles).

If V be volume at 0° C.

Then volume at t_1° C. $= V\left(1 + \dfrac{t_1}{273}\right) = V_1$, or using a for $\frac{1}{273}$

$$\text{volume at } t_1^\circ \text{ C.} = V(1 + at_1) = V_1$$
$$\text{also volume at } t_2^\circ \text{ C.} = V(1 + at_2) = V_2$$
$$i.e.\ V_1 = V(1 + at_1)$$
$$V_2 = V(1 + at_2)$$

that is—

$$V_1 : V_2 :: V(1 + at_1) : V(1 + at_2)$$
$$\text{or } V_1 : V_2 :: (1 + at_1) : (1 + at_2)$$
$$\text{or } \frac{V_1}{1 + at_1} = \frac{V_2}{1 + at_2} \qquad \text{........................(2)}$$

We wish to combine equations (1) and (2) obtained from the two given laws.

That is, having obtained—

1. A relation between the volume and the pressure, when the temperature is constant, viz.—

$$\mathrm{VP} = vp$$

2. A relation between the volume and the temperature, when the pressure is constant

$$\frac{\mathrm{V}_1}{1 + at_1} = \frac{\mathrm{V}_2}{1 + at_2}$$

We need—

3. A relation between the volumes when both pressure and temperature change.

Before endeavouring to combine the equations obtained, we will illustrate by two or three examples.

Example 1.—If 50 cubic inches of air at 0° C. and 38 inches pressure is raised in temperature to 25° C., the pressure falling to $36\frac{1}{4}$ inches, what volume will the air now occupy?

Suppose the change of volume to be produced in two processes: (1) the pressure changing from 38 inches to 36·5 inches, whilst the temperature remains at 0° C.; (2) the pressure remaining unchanged, whilst the temperature rises from 0° C. to 25° C.

(1) Let V^1 be the new volume, whilst the pressure changes from 38 inches to 36 inches, we have—

$$50 \times 38 \text{ cubic inches} = \mathrm{V}^1 \times 36{\cdot}5$$

$$\text{from which } \mathrm{V}^1 = \frac{50 \times 38}{36{\cdot}5} \text{ cubic inches}$$

(2) The pressure remains constant; the temperature rises from 0° C. to 25° C.

$$\therefore \frac{\mathrm{V}^1}{1 + at_1} = \frac{\text{final volume}}{1 + at_2}$$

$$\therefore \frac{\frac{50 \times 38}{36{\cdot}5} \text{ cubic inches}}{1 + \frac{1}{273} \times 0} = \frac{\text{final volume}}{1 + \frac{25}{273}}$$

$$\therefore \text{final volume} = \text{volume required}$$

$$= \frac{\frac{50 \times 38}{36{\cdot}25} \times (1 + \frac{25}{273})}{1 + \frac{1}{273} \times 0} \text{ cubic inches}$$

which can readily be worked out.

Example 2.—If a gas measures 25 cubic inches at 14° C. under a pressure of 35 inches, what volume will it occupy at 0° C. under a pressure of 76 inches?

(1) Pressure changes, temperature remains constant—

$$25 \times 35 \text{ cubic inches} = V^1 \times 76$$

$$\therefore V^1 = \frac{25 \times 35}{76}$$

(2) Temperature changes, pressure remains constant.

$$\frac{V^1}{1 + at_1} = \frac{\text{final volume}}{1 + at_2}$$

$$\frac{\frac{25 \times 35}{76}}{1 + \frac{14}{273}} = \frac{\text{final volume}}{1 + \frac{1}{273} \times 0}$$

$\therefore$ final volume = volume required

$$= \frac{25 \times 35}{76} \times \frac{273}{287} \text{ cubic inches}$$

$$= \frac{25 \times 5}{76} \times \frac{273}{41} \text{ cubic inches}$$

$$= \frac{125 \times 273}{76 \times 41} \text{ cubic inches} = 19{\cdot}3 \text{ cubic inches}$$

Suppose the change from volume V_1 at pressure P and temperature t_1 to volume V_2 at pressure p and temperature t_2 to be brought about in two separate steps.

(1) By change of pressure from P to p, t_1 being constant. In this case, if v^1 be the volume, we have by Boyle's law—

$$V_1P = v^1p \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

(2) By change of temperature from t_1 to t_2, pressure p remaining the same. Then by Charles' law—

$$\frac{v^1}{1 + at_1} = \frac{V_2}{1 + at_2} \quad \ldots\ldots\ldots\ldots\ldots(4)$$

$$\text{But from (3) } v^1 = \frac{V_1P}{p}$$

Substitute this value in (4), we get—

$$\frac{\frac{V_1P}{p}}{1 + at_1} = \frac{V_2}{1 + at_2}, \text{ or finally } \frac{V_1P}{1 + at_1} = \frac{V_2p}{1 + at_2}$$

ABSOLUTE ZERO AND ABSOLUTE TEMPERATURES.

The volume of a gas, whatever pressure may be, is increased $\frac{1}{273}$ of its volume for every increase of 1° C. of temperature, always reckoning from 0° C. That is—

1 volume at 0° C. becomes $(1 + \frac{1}{273})$ volume at 1° C.
1 " 0° C. " $(1 + \frac{2}{273})$ " 2° C.

Now, if instead of increasing the temperature, the temperature be decreased, we get as follows—

1 volume at 0° C. becomes $(1 - \frac{1}{273})$ volume at − 1° C.
1 " 0° C. " $(1 - \frac{2}{273})$ " − 2° C.

and so on.

If this process were continued, we should find a corresponding decrease of volume, and if the temperature −273° C. could be reached, we should then have—

1 volume at 0° C. becomes $(1 - \frac{273}{273})$ volume at −273° C.

in other words—

1 volume at 0° becomes volume 0 at − 273° C.

i.e. the contraction in volume would equal the volume itself, and the volume would cease to exist.

In all probability the gas would undergo some change before this temperature were reached, *e.g.* the gas would most likely become a liquid.

The point −273° on the Centigrade scale is called the *absolute zero of temperature*, and all temperatures reckoned from that point are called *absolute temperatures*. As this point is 273 below 0° C., the absolute temperatures are obtained from those of the Centigrade scale by adding 273 to them. Thus − 6° C. is $(-6 + 273°)$ on the absolute scale,

i.e. − 6° C. = 267° absolute scale

or, as a second example—

+ 12° C. is $(12 + 273)°$ on the absolute scale,
i.e. + 12° C. = 285° absolute scale.

To recapitulate briefly, we have seen that—

(1) The volume of a gas varies *inversely* proportional as the pressure.

(2) The volume of a gas varies directly as the temperature;

i.e. if the pressure increases the volume decreases, whereas if the temperature increases the volume increases.

i.e. if V_1, P_1, T_1, be the volume, pressure, and temperature respectively at a given time

i.e. if V_2, P_2, T_2, be the volume, pressure, and temperature at a second given time

i.e. if V_3, P_3, T_3, be the volume, pressure, and temperature at a third given time

we have $\frac{V_1P_1}{T_1} = \frac{V_2P_2}{T_2} = \frac{V_3P_3}{T_3}$ = pressure and temperature = a constant quantity = C. say,

$$\text{or generally } \frac{VP}{T} = C$$

$\therefore$ the formula found above can be thus written $VP = CT$.

The use and meaning of this result will be best seen by means of a few examples.

Example 1.—A certain quantity of gas is contained in a vessel, whose volume is one cubic foot, and its temperature is 20° C. If in any way, *e.g.* by pressing down a piston, its volume is changed to 1000 cubic inches, and its temperature raised to 30° C., find the ratio of the pressure of the gas in its former state to its pressure in its latter state. (N.B.—The coefficient of expansion is 0·00366.)

[*S. & A.*, 1892

In the example given—

V_1 = 1 cubic foot = 1728 cubic inches
= original volume, or volume at first temperature and pressure
V_2 = 1000 cubic inches = volume at second temperature and pressure
t_1 = 20° C. first temperature
t_2 = 30° C. second temperature
a = coefficient of expansion = 0·00366 (= $\frac{1}{273}$).

We are asked to find the ratio of—

P to p.

Substitute the values given in the formula found. We have—

$$\frac{1728 \times P}{1 + \cdot 00366 \times 20} = \frac{1000 \times p}{1 + \cdot 00366 \times 30}$$

$$\therefore \frac{P}{p} = \frac{1000}{1728} \times \left(\frac{1 + \cdot 00366 \times 20}{1 + \cdot 00366 \times 30}\right)$$

$$= \frac{125}{216} \times \frac{1 + \cdot 0732}{1 + \cdot 1098}$$

$$= \frac{125}{216} \times \frac{1 \cdot 0732}{1 \cdot 1098}$$

which can readily be worked out.

Example 2.—A cubic vessel, whose edge is 1 foot, is made air-tight when the barometer stands at 30 inches, and the temperature of the air is 15° C.; if the temperature of the air is raised to 60° C., what is the increase of the pressure on the contained air on each face of the cube? (N.B.—The coefficient of expansion is 0·003665, and a cubic inch of mercury may be taken to weigh half a pound.)
[*S. & A.*, 1893.

In the example given the pressure on each face of the cube varies as the temperature. Therefore with an increased temperature, an increased pressure will be the result. This will be seen at once, if we remember that the effect of the temperature on the air inside the cube is to cause a tendency to expand.

The pressure at first will be proportional to $(273 + 15)$, *i.e.* to 288.

The pressure in the second case will be proportional to $(273 + 60)$, *i.e.* to 333. This will be seen from the ratio—

$$\frac{V_1}{1 + at_1} : \frac{V_2}{1 + at_2}$$

$$\text{or } \frac{V_1}{1 + \frac{15}{273}} : \frac{V_2}{1 + \frac{60}{273}}$$

$$\frac{V_1}{288} : \frac{V_2}{333}$$

As the air cannot expand, the two pressures will be in the following proportion—

$$V_1 : V_2 :: 288 : 333$$

for coefficient of expansion is given as $0·003665 = \frac{1}{273}$

$\therefore$ increase of pressure is obtained from—

$(30 \times \frac{333}{288} - 30) \times$ area pressed $\times$ weight of cubic inch of mercury, but area pressed = one face of cube = 1 square foot = 144 square inches. Weight of cubic inch of mercury = $\frac{1}{2}$ lb.

$$\therefore \text{ Increase of pressure} = 30\left(\tfrac{333}{288} - 1\right) \times 144 \times \tfrac{1}{2} \text{ lb.}$$
$$= 30 \times \left(\frac{333 - 288}{288}\right) \times 72 \text{ lbs.}$$
$$= \tfrac{30}{4} \times (45) \text{ lbs.} = \tfrac{15}{2} \times 45 \text{ lbs.}$$
$$= 337\tfrac{1}{2} \text{ lbs.}$$

To verify our result—

$$\text{1st pressure} = 30 \times 144 \times \tfrac{1}{2} \text{ lb.}$$
$$= 30 \times 72 \text{ lbs.}$$
$$= 2160 \text{ lbs.}$$
$$\text{2nd pressure} = \tfrac{333}{288} \times 30 \times 144 \times \tfrac{1}{2} \text{ lb.}$$
$$= \frac{333 \times 30}{4} \text{ lbs.}$$
$$= \tfrac{9990}{4} \text{ lbs.}$$
$$= 2497\tfrac{1}{2} \text{ lbs.}$$
$$\therefore \text{ Increase of pressure} = (2497\tfrac{1}{2} - 2160) \text{ lbs.}$$
$$= 337\tfrac{1}{2} \text{ lbs.}$$

Example 3.—It may be assumed that 100 cubic inches of air weigh 33 grains when the barometer stands at 30 inches; find the weight of the air that gets out of a room, when the barometer falls from 30 inches to 29·5 inches, the dimensions of the room being 30 feet long, 20 feet wide, and 15 feet high.

[*S. & A.*, 1891

State the physical principle that justifies your calculation.

The volume of a gas varies inversely as the pressure, *i.e.* if the pressure be increased the volume decreases, and if the pressure be decreased the volume increases. This is the physical principle upon which we must base our calculation.

The volume at first is 30 × 20 × 15 cubic feet. By the principle stated—

$$V_1 : V_2 :: p_2 : p_1$$

The pressure is decreased from 30 inches to 29·5 inches.

$$\therefore V_1 : V_2 :: 29{\cdot}5 : 30$$

$$\text{or } 30 \times 20 \times 15 \text{ cubic feet} : V_2 :: 29{\cdot}5 : 30$$

$$\text{or } V_2 = \frac{30 \times 20 \times 15 \times 30}{29{\cdot}5} \text{ cubic feet}$$

$$= \frac{6 \times 20 \times 15 \times 30}{5{\cdot}9} \text{ cubic feet.}$$

The volume which escapes is—

$$V_2 - V_1 = \left(\frac{6 \times 20 \times 15 \times 30}{5{\cdot}9} - 30 \times 20 \times 15\right) \text{c. ft.}$$

$$= 30 \times 20 \times 15 \left(\frac{6}{5{\cdot}9} - 1\right) \text{cubic feet.}$$

$$= \frac{30 \times 20 \times 15}{5{\cdot}9} \times {\cdot}1 \text{ cubic foot.}$$

$$= \frac{30 \times 20 \times 15}{5{\cdot}9} \times \frac{1}{10} \text{ c. ft.} = \frac{30 \times 2 \times 15}{5{\cdot}9} \text{ c. ft.}$$

But we are asked to find the weight of this volume. We must remember that under the altered conditions the weight of 100 cubic inches will be—

$$\frac{33 \times 29{\cdot}5}{30} \text{ grains}$$

$$\therefore \text{Weight required} = \frac{\cancel{30} \times 2 \times 15}{\cancel{5{\cdot}9}} \times 1728 \times \frac{33 \times \overset{5}{\cancel{29{\cdot}5}}}{\cancel{30}} \times \frac{1}{100} \text{ grs.}$$

$$= \frac{\cancel{2} \times \overset{3}{\cancel{15}} \times \overset{864}{\cancel{1728}} \times 33 \times \cancel{5}}{\underset{2}{\cancel{\underset{10}{\cancel{100}}}}} \text{ grains}$$

$$= 3 \times 864 \times 33 \text{ grains}$$

from which final answer can be obtained.

EXAMPLES ON CHAPTER XX.

1. What is the coefficient of expansion of all gases for 1° C.? and for 1° F.?

2. Enunciate Dalton's law, and also Charles' law. Explain each letter in the formula—

$$V : V^1 = T : T^1.$$

3. Combine Boyle's law and Charles' law into one formula, and explain each letter. Enunciate the compound law in words.

4. State exactly what is meant by each of the letters in the formula for gases, viz.—

$$\frac{Vp}{1 + at} = \frac{VP}{1 + aT}$$

State carefully the properties of gases embodied in the formula. [*S. & A.*

5. Supposing the pressure to remain constant, find the volume of 91 cubic inches of hydrogen when heated from 0° to 50° C.

6. If the temperature be changed from 20° to −20° C., find what 500 cubic inches at the former temperature would measure at the latter.

7. What is the increase in volume of a cubic foot of air when the temperature is raised from 51° to 71° F., pressure remaining constant?

8. A gas occupies 117·9 cub. ins. at 9·9° C., and 700·9 millimeters pressure. Find its volume at 0° C. and 1 meter pressure.

9. A quantity of air at a temperature of 15·6° C. has a volume of 4 cub. ft. under a pressure of 12 lbs. to the square inch, what will be its volume at a temperature of 48·7° C., and under a pressure of 14 lbs. to the square inch?

10. A quantity of air which occupies 26 cub. cms. at a temperature of 59° F., and under 29 ins. pressure, is raised to 68° F. and pressure 50 ins. Find the volume of air under the new conditions.

11. A certain quantity of gas is contained in a vessel whose volume is 1 cub. ft., and its temperature is 20° C. If in any way its volume is changed to 1000 cub. ins., and its temperature raised to 30° C., find the ratio of the pressure of the gas in its former state to its pressure in the last ($a = \frac{1}{273}$). [*S. & A.*

12. What is meant by the "absolute zero"? Could such a state of things be realized? Why?

13. How do you arrive at the number −273° C. for the absolute zero? What is the use of this?

14. The temperature of a gas is put down as 14° C. What is its absolute temperature?

15. What is the absolute temperature corresponding to −14° C.?

16. Where is the absolute zero on the F. scale? What are the absolute temperatures corresponding with 32° F., ·6° F., and −40° F.?

CHAPTER XXI.

PRESSURE OF THE ATMOSPHERE.

WE have spoken of the pressure of the atmosphere as being 15 lbs. on the square inch. One experiment to show this is most important and interesting. It is due to Torricelli, who first performed it in the year 1643. It requires very little apparatus, and can be easily done. Take a straight piece of ordinary glass tubing about a yard long. Let one end be closed, and the other open. Now, by holding the tube obliquely, it can be filled with mercury. Let the open end be covered by the thumb, and the tube inverted. Place the open end under the surface of the mercury contained in a dish, say a saucer; hold the tube vertically.

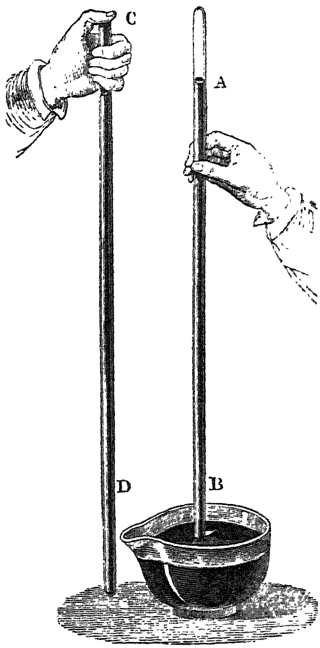


FIG. 106.

At once we see the mercury in the tube sink until the column in the tube is about 30 inches long measured from the surface of the mercury in the dish. Why is this? To answer this question, let us examine the forces acting on the open end of the tube. These are two, viz.:—

1. The pressure of the atmosphere on the mercury in the tube, which acts vertically upwards, transmitted by the mercury in the vessel.

2. The weight of the mercury in the tube, vertically downwards.

It will be seen that there is no pressure vertically downwards due to the atmosphere on the surface of the mercury in the tube, since the tube is closed at the upper end. The mercury in the tube being at

rest, the two forces mentioned must be in equilibrium; as they act in opposite directions in the same straight line, they must be equal. We have, therefore, arrived at the fact that the pressure of the atmosphere is equal to the weight of the column of mercury in the tube. It will be found to be as stated, viz. about 15 lbs. to the square inch. Suppose we had used water instead of mercury, what would have been the length of the column? The specific gravity of mercury, which is 13·6, will help us to answer this question.

Mercury is 13·6 times heavier than water, therefore the column of water must be 13·6 times longer than the column of mercury;

$$\begin{aligned}\therefore \text{length of column of water} &= 13{\cdot}6 \times 30 \text{ inches}\\ &= \frac{13{\cdot}6 \times 30}{12} \text{ feet}\\ &= 34 \text{ feet.}\end{aligned}$$

We shall see how helpful this result is when we come to describe the pump, etc.

The space above the column of mercury and below the closed end of the tube is called the Torricellian vacuum.

Barometer.—In our last experiment we have practically constructed a barometer, which is really an instrument for measuring the pressure of the atmosphere. We need give very little more description, except to point out that the two essentials are—

1. That the mercury in the tube should be open to the upward pressure of the atmosphere from the bottom. This is effected by the cistern being uncovered.

2. The mercury in the tube should be protected from the downward pressure of the atmosphere. This is effected by the upper end of the tube being closed.

Various forms of the barometer are in use. Thus there is the one with the straight tube, as shown in Fig. 107, where the alterations in the height of the column of mercury are actually seen.

We have also the siphon barometer, where the tube is bent, as in Fig. 108, one arm being longer than the other. The short arm, being open, acts as the cistern. The other arm is closed—the difference in the heights of the two columns being the length of the column whose alterations are noticed. Again there is the wheel barometer (Fig. 109). The difference here consists in the addition of a weight which floats in the

shorter arm of the syphon barometer. To this weight a cord is attached which, passing over a pulley, carries a second weight

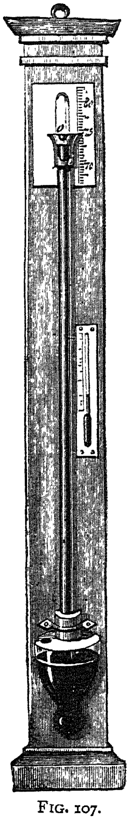


FIG. 107.

FIG. 108.

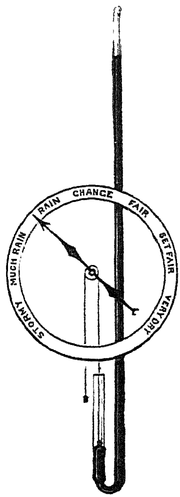


FIG. 109.

not quite so heavy as the first. The pulley carries an index which moves round a graduated circle.

As will be seen, the liquids used may be different, but mercury is the one in most common use. We must again point out that the barometer really only measures the pressure of the atmosphere, *i.e.* as the pressure of the atmosphere increases the column of mercury is lengthened, and *vice versâ.*

The instrument has come to be used as a weather-indicator, because it has been found by a large number of observations that a change in the pressure of the atmosphere, shown by an alteration in the length of the column, predicts a change in the state of the weather. If the barometer sinks, experience tells us that probably rain will come, etc.

This instrument is also useful in determining the difference in the heights of places.

We cannot discuss this question fully, but it will be seen that, as the barometer measures the pressure of the atmosphere, if the height of the column of air representing the atmosphere be diminished—that is, if the barometer be lifted from one station to another, say through a height of 1000 feet —we should expect the barometer to sink. This is really the case, and therefore this fact helps us in the determination of heights.

For standard measurements the length of the mercury column is taken as that at the level of the sea.

Example 1.—A cylinder is fitted with a piston, which works in it easily and air-tight. When the water barometer is at 33 feet the piston is one foot above the bottom of the cylinder. If the cylinder were sunk slowly in water to a depth of 44 feet, how high would the piston be above the bottom of the cylinder?

[*S. & A.*, 1883.

This is a question upon the law of Mariotte. We can solve it by remembering that as the pressure increases the volume decreases. When the pressure is that of a column of water 33 feet long the volume is represented by 1 foot.

Therefore, when this column is increased by 44 feet, we have the proportion—

1 foot : volume required :: 44 + 33 : 33
1 foot : " " :: 77 : 33
" " :: 4 : 3

Therefore volume required is represented by $\frac{3}{7}$ foot, *i.e.* the piston in this case is $\frac{3}{7}$ foot above the bottom of the cylinder.

Example 2.—A barometer stands at 30 inches. The vacuum above the mercury being perfect, the area of the cross section of the tube is a quarter of a square inch. If a quarter of a cubic inch of the external air be allowed to get into the barometer, and the mercury is found to fall 4 inches, what was the volume of the original vacuum? [*S. & A.*, 1881.

This is an interesting and important problem. Using the

figure, we have A B = 30 inches; also the section of the tube $\frac{1}{4}$ square inch.

Let the mercury fall to P when the air is introduced, so that B P is 4 inches. The air, which originally occupied $\frac{1}{4}$ cubic inch, now occupies the space D P; *i.e.* if the length of B D = x inches, it occupies—

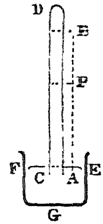


FIG. 110.

$$(x + 4) \times \tfrac{1}{4} \text{ cubic inch} = \frac{x+4}{4} \text{ cubic inches}$$

Originally this air was subjected to a pressure equal to 30 inches of mercury.

We have now to find the new pressure to which it is subjected, which is the pressure of the atmosphere minus the pressure equal to the weight of the column of mercury, whose height is P A, which is equal to 26 inches; *i.e.* it is subjected to a pressure equal to (30 − 26) inches = 4 inches.

Therefore, by Mariotte's law, we have the proportion—

Original volume of admitted air : new volume of admitted air :: new pressure : original pressure;

$$\therefore \tfrac{1}{4} : \frac{x+4}{4} :: 4 : 30$$

$$1 : x+4 :: 4 : 30$$

$$4x + 16 = 30$$

$$4x = 14$$

$$x = 3\tfrac{1}{2}$$

Therefore the length of B D is $3\frac{1}{4}$ inches, and the volume of the original vacuum—

$$= (3\tfrac{1}{2} \times \tfrac{1}{4}) \text{ cubic inches}$$
$$= \tfrac{7}{8} \text{ cubic inches.}$$

In the barometer tube the space above the mercury and below the closed end of the tube is called a Torricellian *vacuum.* It is not a *perfect* vacuum, however, for this space is filled with vapour of mercury. Why this space is so occupied will be seen when we remember that at ordinary temperatures, and at the ordinary atmospheric pressure vapour is being formed slowly at the surface of such liquids as ether, water, mercury, etc. This process is called evaporation, and the statement made can be easily tested by exposing a saucer full of water to the atmosphere for some time. At the end of the time it will be seen that the volume of water has sensibly decreased, and were the water allowed to stand longer it would at length "dry up," *i.e.* become *evaporated* or changed into vapour. This is a slow process, because of the resistance of the pressure of the atmosphere. Take

away this resistance, and the evaporation will be instantaneous. This is exactly what has been done in the experiment described at the commencement of the chapter. The resistance of the pressure of the atmosphere has been removed, and vapour of mercury is at once formed. This vapour filling the space, has a slight effect upon the height of the mercury in the tube. If other liquids were used, we should find that each "vacuum-vapour" would produce its own effect upon the height of the column of liquid in the tube, which effect would differ from the corresponding effects of other liquid vapours.

From the above a definition of the word "vapour" may be obtained: it is the gas into which certain liquids, called volatile liquids, such as ether, alcohol, mercury, water, are changed by the absorption of heat.

THE SIPHON.—This is an instrument the working of which depends entirely upon the pressure of the atmosphere. In its simplest form it is a bent tube having one leg shorter than the other. It is used for emptying casks, vessels, etc., of the liquids contained therein.

To start its action, all that is needed is that it should be first filled with the liquid and then inverted, so that the shorter leg is placed below the surface of the liquid in the cask.

The theory of its action can be seen by reference to the following figure.

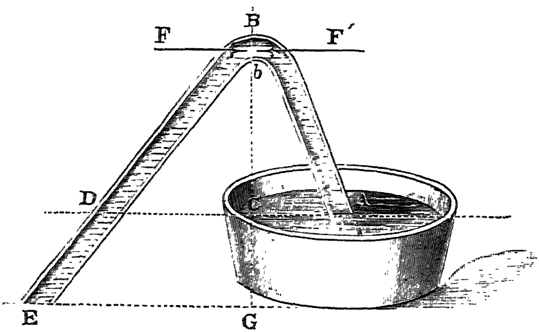


FIG. 111.

Let the surface of the liquid in the vessel be at A. Draw a horizontal line A C D so that D is a point in the longer leg in the same horizontal plane as A. Examine the liquid at the

section B *b* of the siphon. This is acted upon by two forces, F and F′, in opposite directions.

1. F′, the pressure on the side towards A B
2. F, " " " E B

If these two forces were equal, the liquid would be at rest and no motion would ensue.

Let us take each force separately, and, to make our reasoning clear, let us suppose the vessel contains milk of specific gravity 1·02.

We measure the two vertical heights B C, B G;

B C we will suppose to be 18 inches long,
B G " " 24 "

Taking the height of the water barometer to be 34 feet, we know that if milk had been used in the place of water the column would have been $\frac{34}{1.02}$ feet $= 33\frac{1}{3}$ feet.

We can now proceed with our investigation.

1. The force F′ is equal to the pressure of the atmosphere minus the pressure equal to the weight of the column of milk of height B C, *i.e.* force F′ is equal to a pressure equivalent to the weight of a column of milk of height $(33\frac{1}{3} - 1\frac{1}{2})$ feet. Therefore force F′ may be taken as the weight of a column of milk of height $31\frac{5}{6}$ feet.

2. The force F is equal to the pressure of the atmosphere minus the pressure equal to the weight of the column of milk of height B G, *i.e.* force F is equal to a pressure equivalent to the weight of a column of milk of height $(33\frac{1}{3} - 2)$ feet. Therefore force F may be taken as the weight of a column of milk of height $31\frac{1}{3}$ feet. Therefore we have the result that force F′ is greater than force F by the difference between the weights of the two columns $31\frac{5}{6}$ feet high and $31\frac{1}{3}$ feet high, *i.e.* by a column $\frac{1}{2}$ foot high.

From this it will be seen that the action of the instrument depends upon the two heights B C and B G.

If we had used general terms, and considered the pressure of the atmosphere to be equal to H inches of mercury—

Force F′ would have been equal to the weight of H inches of mercury minus B C inches of the liquid;

Force F would have been equal to the weight of H inches of mercury minus B G inches of the liquid:

i.e. force F′ is greater than force F by (B G − B C) inches of the liquid.

If such be the case, motion will take place from A B towards B E.

From what has been said, we at once see that if a continuous flow is to be maintained we must have the following conditions—

1. The shorter leg must be kept underneath the surface of the liquid in the vessel.

2. B C must not be greater than H, *i.e.* the length of the shorter leg must not be greater than the length of the column of liquid whose height is equal to the pressure of the atmosphere. If B C were greater than H, the quantity H − B C would be negative, and we should have a flow back towards A, the surface of the liquid. This point must be noticed as the surface of the liquid in the vessel continues to sink.

3. The length B G must be greater than the length B C.

The Suction or Common Pump.—This machine depends for its action upon the pressure of the atmosphere. This pressure has been found to be equal to the weight of a column of water 34 feet high. To show how this helps us to understand the theory of the machine, we will give its essential parts. With reference to the figure they are—

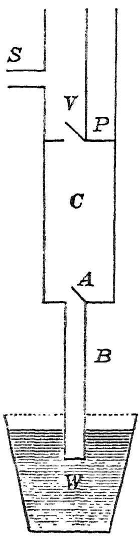


Fig. 112.

1. Cylinder C;
2. Pipe B, leading to well W;
3. P, air-tight piston fitting in C;
4. Valve V in P, opening upwards;
5. Valve A at bottom of cylinder, opening upwards;
6. Spout S.

First let us suppose the piston P to be at the bottom of the cylinder, and the pipe B to be full of air. We have now two equal forces acting on the piston P and on the valve A, viz.:

1. The pressure of the atmosphere, acting downwards on the piston and on the valve A.

2. The pressure of the atmosphere, acting upwards, transmitted by the water in the well along the pipe B.

These two forces are equal, and therefore the air in the

cylinder and pipe as well as the water in the well are at rest.

Let the piston be raised; we have now different conditions: the valve A is now only acted upon by one force, viz: the pressure of the atmosphere below it. This pressure forces up the water from the well into the pipe, and the air from the pipe into the cylinder.

At the next motion of the piston downwards the valve A closes and V opens; the air in the cylinder below V is thus forced through V as the piston descends, until P reaches the bottom of the cylinder.

At the next movement of P upwards the valve A is acted on only by the pressure of the atmosphere below it. This pressure forces the water from the pipe into the cylinder. At the next stroke P is moved downwards, V opens, A closes, and the water passes above P from below. At the next stroke of P upwards the water is lifted up and so conducted to the spout.

The force required to work the pump will be obtained from the lengths of the parts into which the piston divides the distance from the surface of the water at the spout to the surface of the water in the well.

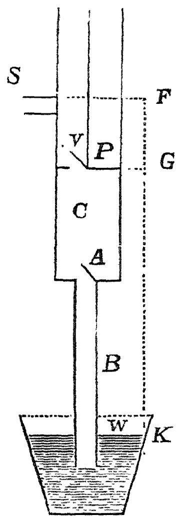


FIG. 113.

Thus, taking Fig. 113, and remembering that the pressure of the atmosphere is equal to the weight of a column of water 34 feet in height, we know that the pressure on the top of the piston is equal to the weight of a column of water $(34 + FG)$ feet high and A square feet in section, where A equals area of the piston.

For simplicity, suppose that this area is 1 square foot: then the pressure on the top of the piston is equal to the weight of a column of water $(34 + FG)$ feet in height.

Also the pressure on the bottom of the piston is equal to the weight of a column of water $(34 - GK)$ feet in height.

Therefore the force we need must just exceed the difference between these two, *i.e.* it must be just greater than the weight of a column of water $\{(34 + FG) - (34 - GK)\}$ feet in height and 1 square

foot in section; *i.e.* it must be just greater than the weight of the column (F G + G K) feet in height; *i.e.* it must be just greater than the weight of the column F K feet in height.

Thus the force required depends upon the distance of the spout from the surface of the water in the well. If this distance be equal to 30 feet, and the section 1 square foot, then the force needed must just exceed the weight of 30 cubic feet of water, *i.e.* the force must just exceed 30 × 1000 ozs. Therefore the force needed must be just greater than 1875 lbs.

It will be seen that another condition for working the pump is that the piston must not be placed more than 34 feet above the surface of the water in the well.

The arrangement by which water can be raised to a height greater than 34 feet is called the FORCE PUMP.

Here we have several constructions different from those in the suction pump. Referring to the figure, we notice that the piston of the suction pump is replaced by the plunger P, which is solid, having no valve. The spout is replaced by a pipe leading upwards from the cylinder, placed near the bottom of the cylinder. The bottom of this pipe is covered by a valve V opening upwards. To examine the action of the force-pump, suppose the plunger P to be at the bottom of the cylinder. As the plunger rises the valve A opens, and water enters the cylinder because of reasons already given. When the plunger descends A closes, V opens, and the water is forced through V into the pipe. When the plunger rises on the next stroke A opens, but V is kept closed by the weight of the water above it in the pipe. On the next descent of the plunger more water is forced into the pipe, and thus the water can be forced to any height. An addition is frequently made to the above apparatus to secure a continuous flow instead of a succession of jerks. It consists of a chamber filled with air, continuous with the pipe from the pump. A second pipe leads from this vessel outwards.

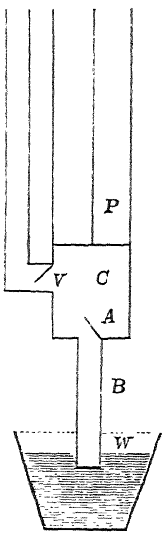


FIG. 114.

As water is forced into this chamber the air within it

becomes compressed, and thus by the elasticity of this compressed air a continuous flow is maintained.

The fire-engine consists of two force-pumps connected by such a chamber; these pumps, worked by the lever, are so arranged that as one plunger descends, closing the valve below it, the other ascends and opens the valve which leads to the water in the well.

EXAMPLES ON THE PUMP.

Example 1.—Describe briefly the suction pump and its action. The diameter of the piston is 4 inches, and the spout is 20 feet above the surface of the water in the well; find the force along the piston rod required to work the pump. Explain how the calculation of the force is justified. [*S. & A.*, 1892.

From the explanation of the suction pump given in the text, we have—

Force required to work the pump must be just greater than the weight of a column of water $\{(34 + F\,G) - (34 - G\,K)\}$ feet in height and section, whose diameter is 4 inches.

Therefore force required is just greater than weight of column of water $(F\,G + G\,K)$ feet in height (see Fig. 113) and $\pi 2^2$ square inches in section.

$$\text{But } F\,G + G\,K = 20 \text{ feet}$$

therefore column is 20 feet in height and $\frac{22}{7} \times 4$ square inches in section.

Therefore force required must be just greater than

$$20 \times \tfrac{22}{7} \times \tfrac{4}{144} \times 1000 \text{ ozs.}$$

(for a cubic foot of water weighs 1000 ozs.).

Therefore force required must be just greater than

$$\frac{20 \times 22}{7 \times 36} \times 1000 \text{ ozs.} = \frac{20 \times 22}{7 \times 9} \times 250 \text{ ozs.}$$

from which final result can readily be obtained.

Example 2.—Explain the action of the forcing-pump, and show in diagram the valves (*a*) when the piston is being raised, (*b*) when it is being forced down. [*S. & A.*, 1893.

If the piston has a cross section of 5 square inches, and the cistern is 40 feet above the piston, find the number of foot-pounds of work done when the piston is forced down through 1·5 feet. The first part of the question can readily be answered from the explanation given in the text.

The numerical part of the answer is as follows:—

Here a column of water of height 1·5 feet and cross section 5 square inches is lifted through 40 feet; as the cistern is 40 feet above the piston.

(1) To find the weight raised—

Weight of column of water 1·5 feet high, and cross-section 5 square inches $= 1\cdot5 \times \frac{5}{144} \times 1000$ ozs.

(as a cubic foot of water weighs 1000 ozs.)

$\therefore$ weight raised $= \frac{3}{2} \times \frac{5}{144} \times \frac{1000}{16}$ lbs.

$$= \frac{1 \times 5 \times 125}{2 \times 48 \times 2} \text{ lbs.}$$

$$= \frac{625}{192} \text{ lbs.}$$

(2) Height through which the weight is raised = 40 feet.

(3) $\therefore$ number of foot-pounds of work done when piston is forced

$$\text{down} = \frac{625 \times 40}{192} = \frac{625 \times 5}{24}$$

$$\therefore \text{ answer } = \frac{3125}{24} \text{ ft.-lbs.} = 130\tfrac{5}{24} \text{ foot-pounds.}$$

Example 3.—Describe briefly and explain the action of the common force-pump. When the piston (or plunger) is lifted, why does the water follow it? [*S. & A.*, 1885.

If the plunger has a cross section of 4 square inches, and works 80 feet below the cistern, with what pressure must it be pushed down in order to force water into the cistern?

It has to lift a column of water 80 feet high, whose cross section is 4 square inches.

Therefore it has to overcome a resistance of

$$80 \times \frac{4}{144} \times \frac{1000}{16} \text{ lbs.}$$

Therefore pressure required must be just greater than

$$\frac{5 \times 4 \times 1000}{144} \text{ lbs.}$$

Therefore pressure required must be just greater than

$$\frac{5 \times 2 \times 125}{9} \text{ lbs.}$$

Therefore answer required is, pressure required must be just greater than $138\frac{8}{9}$ lbs.

The Aneroid Barometer.—The principle upon which this instrument works can be readily understood from the following description of its essential parts.

One of its forms consists of a cylindrical metal box, from which the air has been withdrawn.

The pressure of the atmosphere is what is really measured, and this can be done by making the lid of the box of some

substance which will readily yield to the alteration in this pressure. The lid is thus made of thin corrugated elastic metal.

If we remember that because the air has been withdrawn from the inside there is no internal pressure, it will easily be understood that when the pressure of the atmosphere increases in the slightest degree the lid is forced inwards. When the pressure decreases the elasticity of the lid, aided by a spring, causes the lid to move outwards. These motions are indicated on a dial by means of an index which is moved by a number of connected levers. The advantages of this barometer are: (1) it is portable; (2) the delicacy with which it indicates the slightest difference of pressure. These two points cause it to be much used for determining the difference of heights of stations, the heights of mountains, etc.

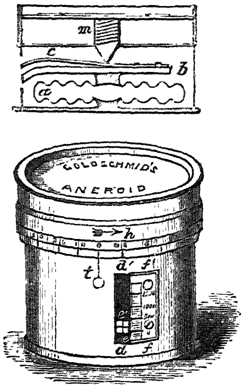


FIG. 115

THE COMPRESSED AIR MANOMETER.—In speaking of Mariotte's law, we have said that the volume of a gas is inversely proportional to the pressure brought to bear upon it; *i.e.* a certain volume of air (say) under the pressure equal to the weight of a column of mercury 30 inches high, *i.e.* a pressure of one atmosphere, becomes half that volume if a pressure equal to the weight of a column of mercury, 60 inches high, or of two atmospheres, be brought to bear upon it. The student will already have seen that the elasticity of the gas increases as the pressure increases, or as the volume a certain quantity of gas occupies is made to decrease; *i.e.* the force with which the gas resists the compression increases proportionally with the pressure because of the elasticity of the gas. To measure this elasticity, or to measure the elasticity of the compressed air or gas, an instrument called a *manometer* is used. The simplest form of manometer can be thus described. It consists of a long narrow tube of uniform bore, closed at A, which dips into the

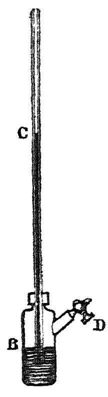


FIG. 116.

mercury contained within a strong vessel. The vessel is so constructed that the air or gas to be measured can be admitted through an opening at D. The elasticity of the air or gas will be measured by the height of the mercury column in the tube. If the mercury stands at about 30 inches, we know from previous work that the air is under a pressure equal to that of the ordinary atmosphere. If air under pressure enters the vessel through the opening D, the height of the column will be increased, the air within A C will be compressed according to Mariotte's law, and we shall know the pressure to which the air entering through D is subject by noticing the length of the column or of A C.

If the mercury stands at 60 inches, then the pressure is equal to that of two ordinary atmospheres, and so on

Example.—Suppose the length of the column of mercury is 6 feet, what pressure in pounds per square inch is the air entering through D subject to?

Pressure per square inch when column is 30 in. = 15 lbs.

" " " 6 ft. $= \frac{15 \times 6 \times 12}{30}$ lbs.

Pressure is 36 lbs. per square inch.

As will be seen, the great inconvenience of this is that the glass tube must be of great length when great pressures are to be measured; for, as 30 inches represents a pressure of one atmosphere, *i.e.* of 15 lbs. on each square inch, we shall need this additional length when the pressure is increased one atmosphere.

An instrument more convenient for such cases is represented in the figure. Its essential parts are (1) a bent tube closed at one end, containing air at the ordinary pressure; (2) a quantity of mercury in the tube which separates the enclosed air from the air or gas to be measured — this is represented by the shaded part of the figure; (3) the other end of the tube communicates with the vessel containing the air or gas whose pressure is to be measured. If the gas is at the ordinary pressure, the mercury will stand at the same horizontal line in both parts of the tube. If the pressure is less than the ordinary, then the mercury will rise towards D and fall from C, because the pressure of the

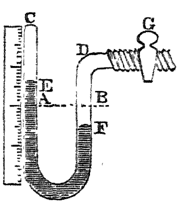


FIG. 117.

enclosed air is greater than the pressure on the other side of the mercury.

If the gas is at a pressure greater than the ordinary, then the mercury rises towards C and falls from D, as will at once appear.

The pressures can be accurately measured by means of a scale attached to the leg C E.

THE AIR-PUMP.—What we have already said in the section on the common pump will help us here.

The air-pump is used to extract air from vessels where air of less density than that of the outside air is needed, or where

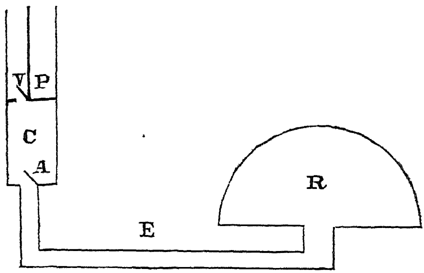


FIG. 118.

an approach to a vacuum is desired. The essential parts are the following, which will be found to be practically the same as those of the suction pump.

1. A cylinder C, leading to the outer air.

2. An air-tight piston P, fitting in C, having a valve V opening outwards.

3. A pipe E, communicating with the cylinder C by means of a second valve A, also opening outwards.

This pipe communicates at the other end with the vessel from which the air is to be extracted. This vessel may be a receiver, R, as in the figure, or a glass cylinder or globe, or piece of glass tubing, which can be made to fit air-tight over the aperture Q.

If the suction pump has been understood, there will be little difficulty in understanding the principle of the action of the air-pump.

1. Let us suppose that the piston P is at the bottom of the cylinder. The pressures now on each side of P are equal.

2. On the piston being raised, say to D, we notice that

the valve V will remain closed because of the external air, and the air which originally filled the receiver R and the pipe E will now occupy not only R and E, but also that part of the cylinder between the valve A and the piston P. This will take place because the pressure at A has been relieved, and therefore the air will expand.

3. On the descent of P the valve A will at once close, and the air within the cylinder below the piston will be compressed until its density is the same as that of the outer air. As P descends further, V will open, and the air within the cylinder will be forced out until the piston again reaches the bottom of the cylinder. On the piston being worked a second time the same operation will be repeated, and more air will be withdrawn, *i.e.* at each double stroke some additional air will be exhausted, and the density of the air within the receiver becomes less and less.

One might at first sight suppose that this operation might be repeated until all the air within R is exhausted, but it is found by experience that on a certain degree of exhaustion being reached no further withdrawal can be obtained.

We will now consider the question of the measure of the degree of exhaustion reached at the end of any given number of strokes.

Let the volume of the receiver and the pipe together be R, that of the available part of the cylinder be C, and the densities of the air within the receiver at the commencement of the 1st, 2nd, 3rd, 4th, 5th . . . nth double strokes be $D, D_1, D_2, D_3, D_4 \ldots D_{n-1}$. Then, as the density of the air is inversely as the volume occupied, *i.e.* as the volume increases the density must decrease, we have to find D_1, the density at the end of the first stroke.

$$D : D_1 :: R + C : R$$

i.e. $$\frac{D}{D_1} = \frac{R + C}{R}$$

because the air originally filling R at the commencement of the first stroke now fills R and C.

Thus, suppose $R = 10$ cubic feet and $C = 1$ cubic foot, then the air originally occupying 10 cubic feet now occupies 11 cubic feet. Therefore the density at the end of the first stroke to density at the beginning of the first stroke :: 10 : 11;

$$\text{or, } \frac{D}{D_1} = \frac{11}{10}$$

Also $\frac{D_1}{D_2} = \frac{R + C}{R}$ for the same reason as before; for, using the dimensions given in the last case, we have the air which occupied 10 cubic feet at the commencement of the second stroke now occupying 11 cubic feet. Therefore the density at end of second stroke to density at beginning of second stroke :: 10 : 11,

$$\text{cr, } \frac{D_1}{D_2} = \frac{11}{10}$$

For the same reason above, we have also—

$$\frac{D_2}{D_3} = \frac{R + C}{R}$$

and so on, until we reach—

$$\frac{D_{n-1}}{D_n} = \frac{R + C}{R}$$

We now have the following set of equations :—

$$\frac{D}{D_1} = \frac{R + C}{R}$$

$$\frac{D_1}{D_2} = \frac{R + C}{R}$$

$$\frac{D_2}{D_3} = \frac{R + C}{R}$$

$$\vdots \qquad \vdots$$

$$\frac{D_{n-1}}{D_n} = \frac{R + C}{R}$$

If we multiply together all the left sides of these equations,

we get $$\frac{D}{D_n}$$

if we multiply together all the right sides, we get

$$\left(\frac{R + C}{R}\right)^n$$

$$\therefore \frac{D}{D_n} = \left(\frac{R + C}{R}\right)^n$$

which, expressed in words, means that the destiny of the air within the receiver at the beginning is to the destiny of the

air within the receiver at the end of the nth double stroke in the ratio $\left(\frac{R+C}{R}\right)^n$

To illustrate, suppose that the volume of R is 5 times that of C, and we want to find the destiny of the air within the receiver at the end of the fifth stroke. We have—

$$\frac{D}{D_5} = \left(\frac{5+1}{5}\right)^5$$

$$\text{or,} \quad \frac{D_5}{D} = \left(\frac{5}{6}\right)^5$$

i.e. the density at the end of the fifth stroke is $\frac{3125}{7776}$ of the density of the air within the receiver at the commencement.

Example.—If the pressure in the receiver of an air-pump were reduced to one-third of the atmospheric pressure in four strokes, to what would it be reduced in six strokes? [*S. & A.*, 1883.

We have to remember that the pressure exerted is proportional to the density.

Therefore we have—

$$\frac{D_4}{D} = \tfrac{1}{3}$$

But

$$\frac{D_4}{D} = \left(\frac{R}{R+C}\right)^4 = \tfrac{1}{3}$$

also

$$\frac{D_6}{D} = \left(\frac{R}{R+C}\right)^6 = \left(\frac{R}{R+C}\right)^4 \times \left(\frac{R}{R+C}\right)^2$$

$$= \tfrac{1}{3} \times \sqrt{\tfrac{1}{3}}$$

$$\therefore \frac{D_6}{D} = \frac{1}{3\sqrt{3}} = \frac{\sqrt{3}}{9} = \frac{1{\cdot}732}{9} = {\cdot}192$$

or the density has been reduced to 0·192 of the atmospheric pressure by six strokes.

To measure the degree of exhaustion produced a contrivance is added which is called the *mercurial gauge.*

It is really a manometer, the air to be measured, instead of being compressed, being at a less pressure than ordinary.

It consists of a glass tube A B C D (Fig. 119), of uniform bore, closed at A and open at D, in such a manner that it can be fitted in the pipe at X. The tube is bent as in the figure; the leg A B is filled with mercury. The pressure of the air in the receiver

at the commencement is able to support this column of mercury. If now the machine be worked, some of the air will be withdrawn at each stroke, and therefore at each succeeding stroke the pressure of the internal air becomes less, and is therefore able to support a shorter column of mercury. We have therefore in this column of mercury an accurate measure of the pressure of the internal air, and therefore of the degree of exhaustion obtained.

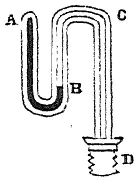


FIG. 119.

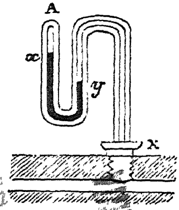


FIG. 120.

We should mention two kinds of air-pump in use.

1. Smeaton's, which is a single-barrelled one, *i.e.* with only one cylinder. The description is akin to what has been given, viz. :—

(1) Cylinder C, with valve A.
(2) Pipe E, leading to receiver.
(3) Piston P, with valve V.

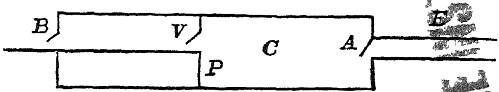


FIG. 121.

It has an additional valve at B, which is useful in relieving the piston P from the pressure of the external air. By this means more complete exhaustion is obtained.

2. Double-barrelled, *i.e.* having two cylinders. Its construction is as follows—

(1) Two cylinders, which communicate by means of a common pipe with one receiver.

(2) Two air-tight pistons, each one fitting in one of the cylinders.

(3) Each piston is fitted with a valve opening outwards.

(4) At the bottom of each cylinder is a valve, also opening outwards.

The cylinders communicate at once with the outer air.

The method of working will be seen from Fig. 122.

The advantage of this machine is that by the alternate

ascent and descent of each piston the downward pressure of the atmosphere is neutralized, and thus less force is needed for its working and the air is exhausted more rapidly.

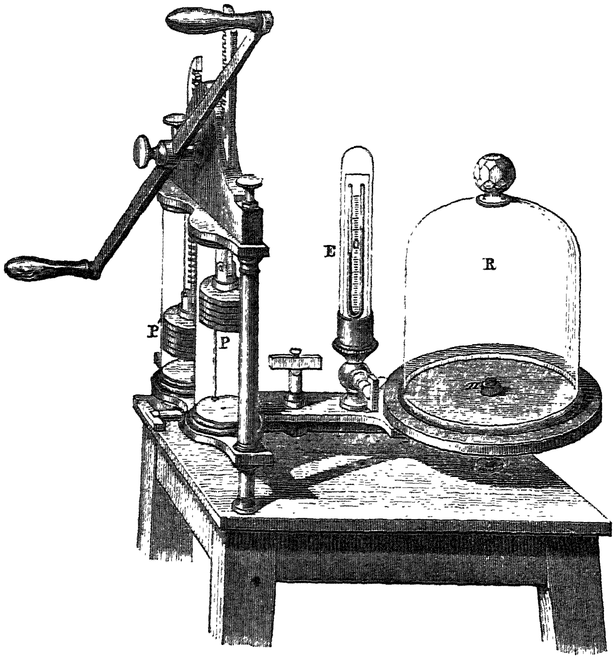


FIG. 122.

EXAMPLES ON THE AIR-PUMP.

Example 1.—Describe briefly the mercurial gauge of an air-pump. If the barometer stands at 29·7 inches, what will the gauge read when the quantity of air withdrawn is 10 times as much as the quantity left in the receiver? [*S. & A.*

Density in receiver at first : density in receiver at observation :: 11 : 1. For 10 volumes have been extracted, and 1 volume left

$$\therefore \quad D_1 : D_n :: 11 : 1.$$

But the original density was measured by 29·7 inches, as indicated by the barometer.

∴ we have $29{\cdot}7 : D_n :: 11 : 1$
or $D_n = \frac{29{\cdot}7}{11} = 2{\cdot}7$ inches

i.e. the gauge will stand at 2·7 inches.

Example 2.—If the pressure in a receiver after 3 strokes be to the atmospheric pressure as 216 : 343, compare the capacities of the receiver and barrel.

Let R be volume of receiver and C the volume of barrel. Then, as in text—

$$\frac{D}{D_n} = \left(\frac{R+C}{R}\right)^n$$

in case given $n = 3$.

$$\therefore \frac{D}{D_3} = \left(\frac{R+C}{R}\right)^3$$

or $$\frac{343}{216} = \left(\frac{R+C}{R}\right)^3$$

or $$\frac{7}{6} = \frac{R+C}{R}$$

or $$7\,R = 6\,R + 6\,C$$

$$\therefore \quad R = 6\,C$$

i.e. volume of receiver is 6 times volume of barrel.

Example 3.—The cylinder of a single-barrelled air-pump has a sectional area of 1 square inch, and the length of the stroke is 4 inches. The pump is attached to a receiver whose capacity is 36 cubic inches. Compare the pressure of the air in the cylinder after 8 complete strokes of the pump with the pressure before commencing the operation.

Volume of cylinder = 1 × 4 cubic inches = 4 cubic inches.
" receiver = 36 " "

∴ density at begin. : density at end of 8 strokes :: $(R + C)^8 : R^8$
or $D : D_8 :: (4 + 36)^8 : 36^8$

or $$\frac{D}{D_8} = \left(\frac{40}{36}\right)^8 = \left(\frac{10}{9}\right)^8$$

$$\therefore \frac{\text{Density at end of 8 strokes}}{\text{Density at commencement}} = \left(\frac{9}{10}\right)^8$$

EXAMPLES.

Barometer.

1. What does this instrument indicate? Compare and contrast it with a thermometer. What makes the mercury rise and fall in these instruments?

2. Describe minutely how you would construct a barometer; state exactly what prevents the mercury from falling out of the tube.

3. Why does the diameter of the barometer tube not interfere with the work of the instrument—supposing it not too narrow?

4. Why should the barometer column fall as we ascend a mountain? Why ascend as we descend a mine? [*S. & A.*

5. Give a rule for the calculation of heights by means of the barometer.

6. What is meant by a "pressure of 30 ins. of mercury;" "a pressure of one atmosphere;" "a pressure of three atmospheres," and so on?

7. Say why the column falls when it is likely to be wet, and why it rises when likely to be dry.

8. Mention as many reasons as you can for the mercury column rising and falling.

9. Does it matter whether you hold the barometer tube in a vertical or oblique position? Which is correct? [*S. & A.*

10. Describe the use and construction of an aneroid barometer.

11. What is meant by the "height of the homogeneous atmosphere"? In what sense can it be spoken of as constant? Show that in this sense it is constant. [*S. & A.*

Suction Pump.

12. Explain the action of the common suction pump, and find the force acting along the piston rod required to work it. [*S. & A.*

13. One foot of the length of the barrel of the pump holds a gallon of water (20 lbs.); at each stroke the piston works through 4 ins.; the spout is 24 ft. above the surface of the water in the well: how many foot-pounds of work are done per stroke? [*S. & A.*

14. The water in a common pump has risen 12 ft. The piston is 3 ins. above the surface of the water in the barrel; find the tension in the piston rod, supposing the area of the barrel to be 8 sq. ins.

15. Why must the height of the lower valve in a common pump above the surface of the water in the well be less than 34 ft.? What arrangement is adopted if you need to raise water from a greater depth than this?

16. A pump is used for raising sea water of specific gravity 1·1. If the mercurial barometer stand at 29·5, what is the greatest length possible for the tube of the pump—reckoning from the surface of sea water to the lower valve? Specific gravity of mercury 13·6.

17. If the area of the piston in the common pump be 6 sq. ins., and the water rise 9 ins. at each stroke, find the power required to work the pump. The pressure of the air is 15 lbs. to the sq. in.

18. The height of the barometer column varies from 28 ins. to 31 ins. What is the corresponding variation in the height to which water can be raised by the common pump? Specific gravity of mercury being 13·6.

19. Describe briefly the common suction pump and its action. The diameter of the piston is 4 ins., and the spout is 20 ft. above the surface of the water in the well; find the force along the piston rod required to work the pump. Explain how the calculation is justified. [*S. & A.*

Force Pump.

20. Describe briefly the action of the common force-pump. If the plunger has a cross section of 8 sq. ins., and works 50 ft. below the cistern, what pressure is required to force it down? [*S. & A.*

21. When the piston or plunger is lifted in a force-pump, why does the water follow it? If the plunger has an area of 4 sq. ins., and works 80 ft. below the cistern, with what pressure must it be pushed down in order to force water into the cistern? [*S. & A.*

Siphon.

22. Explain the action of a siphon. [*S. & A.*

23. Why is the flow more rapid—all other circumstances being the same—the greater the difference in lengths of the legs? [*S. & A.*

24. If a siphon were carried to the top of a high hill, what effect would it have on its action?

25. If a small hole be made in the top of a siphon in operation, what will be the effect?

26. If the hole be made in the legs of the siphon at various points, what will be the effects?

27. If a siphon in operation were placed under a bell jar, and the air gradually exhausted, what would be the effect?

28. The two legs of a siphon are filled with liquids of different density, which do not mix. Each leg is placed in a vessel containing the liquid with which it is filled. The level of the liquids in both vessels is the same. Will there be any motion, and if so, in what direction?

29. If a glass siphon be used for decanting mercury from one vessel to another, how high approximately may the highest point of the siphon be above the level of mercury?

30. What is the greatest height of the siphon tube employed to carry a liquid of specific gravity 2·5 from one vessel to another, when the water barometer stands at 34 ft.?

Air Pump.

31. Describe the air-pump, the single-barrelled pump, or Smeaton's.

32. Explain the advantage gained in the air-pump by the use of two cisterns. [*S. & A.*

33. Describe briefly the action of an air-pump in its simplest form, and explain how the degree of rarefaction produced by a given number of strokes can be approximately calculated. If the pressure were reduced to one-third of the atmospheric pressure in 4 strokes, to what would it be reduced in 6 strokes? [*S. & A.*

34. The cylinder of a single-barrelled air-pump has a sectional area of 1 sq. in., and the length of the stroke is 4 ins. The pump is attached to a receiver of 36 ins. capacity. Compare the pressure of the air in the cylinder after 8 complete strokes with the pressure before commencing

the operation. What conditions limit the exhaustion which an air-pump can effect?

35. If the pressure in a receiver be half the atmospheric pressure after 3 strokes, what will it be after 9 strokes?

36. Describe the mercurial gauge used for measuring the degree of exhaustion of an air-pump. [*S. & A.*

37. The pressure of air in the receiver of an air-pump is 30 ins.; after a few strokes its pressure is reduced to 27 ins.; what part of the original contents of the receiver has been withdrawn? [*S & A.*

38. If the capacity of a receiver of an air-pump is 6 times that of the barrel, and the pressure of the air in the receiver before the first stroke equal to the pressure of 29·4 ins. of mercury, what will be the pressure after the first stroke?

39. The receiver of an air-pump is 5 times as large as the barrel. If the pressure originally be 15 lbs. per square inch, what will it be after 3 strokes of the pump.

40. Describe a common form of condenser. If the contents of the receiver be 10 times that of the barrel, after how many strokes will the pressure of the contained air be to that of the atmosphere as 33 to 30?

Manometer.

41. What is the use of a manometer or pressure gauge? Describe a simple manometer, a compressed-air manometer, a siphon manometer, and a pressure gauge.

42. In the siphon manometer, consisting of a bent tube open at both ends, a liquid of specific gravity 0·85 is used. If a pressure of 20 lbs. per square inch act at one end of the tube, how high will the liquid rise in the other, and what will be the difference of level in the two legs? Atmospheric pressure 15 lbs.

43. In a siphon manometer, 1 sq. in. cross section, mercury (13·6) is used. One end is fitted to a vessel containing gas under a certain pressure; the mercury is then found to have a difference of level of $10\frac{14}{85}$ ins. What is the pressure of the gas per square inch?

44. In the compressed-air manometer, the area of cross section of the tubes is $\frac{1}{8}$ sq. in. When the pressure at the open end is equal to one atmosphere, the mercury in both tubes stands at the same level, at the distance of 8 ins. from the closed end. When the pressure at the open end equals $4\frac{2}{3}$ atmospheres, what will be the distance of the mercury from the closed end, and what is the difference of level in the two tubes? One atmosphere is equal to 30 ins of mercury.

SCIENCE AND ART DEPARTMENT'S EXAMINATION.

SUBJECT VI.

THEORETICAL MECHANICS.

DIVISION II.—FLUIDS.

SYLLABUS.

In the *First Stage* the student is required to make himself acquainted with the axioms and the elementary propositions and formulæ of the science, as simple matters of fact, independently of their formal proof. For instance, he must know the proposition called the parallelogram of forces, as a matter of fact, and must be able to apply his knowledge; but he will not be required to prove the proposition. In like manner he will be required to know what is the metacentre of a floating body, but not to prove the formula for finding its position. In a word, he will be required to make brief exact statements, and to work out easy examples. With perhaps an occasional exception, the examples will be either arithmetical, or capable of being answered by an easy construction drawn to scale. No question involving complicated algebra or geometry will be set. The student should be able to substitute numerical values in an algebraical formula, and to solve a simple equation. He should also have accurate notions of ratios. Every student should bring to the examination a pair of compasses, a scale of equal parts, and a protractor.

The subject of Division II. cannot be understood unless the student has a preliminary acquaintance with the fundamental notions of force and motion, on which all parts of the science are based. These are given in Section 1 of the following syllabus. He should also understand what is meant by the resultant of two or more forces, and that in the case of two or more parallel forces the resultant equals the sum of the forces. He should know what is meant by the centre of gravity of a body, some of its elementary properties, and its position in the case of such bodies as sphere, prism, cylinder, circle, parallelogram.

1. Units of time and distance; measurement of velocity, whether constant or variable; measurement of acceleration, particularly of constant acceleration; acceleration due to gravity; mass or quantity of matter, unit of mass, density, specific gravity (or specific density); momentum; measurement of force; absolute units of force, particularly the poundal or British absolute unit; distinction between the mass and the weight of a body.

2. Definition of energy; distinction between potential and kinetic energy; the work done by a force; absolute units of work, particularly the

foot-poundal; equation of work and energy for constant force acting on a particle.

3. Definition of fluid and liquid; transmission of pressure through a fluid; measurement of pressure at any point of a fluid; surface of a liquid acted on by gravity, and pressure at any point within the liquid; distinction between the whole pressure and the resultant pressure of a liquid on a given surface; magnitude and line of action of resultant pressure in the case of a rectangular area, one edge of which is on the surface of the liquid; also in the case of a body wholly or partly immersed in a liquid.

4. Conditions of equilibrium of a floating body; definition of the metacentre; position of the metacentre in the case of a sphere, and of a cylinder with its axis vertical (the formula $4HM \cdot hs = r^2$ being assumed); stability of floatation.

5. Determination of the specific gravity of insoluble solids, and of liquids, (1) by the balance, (2) by the specific-gravity bottle, (3) by Nicholson's hydrometer; specific gravity of solids lighter than water by the balance; weight of a body in air and in vacuo.

6. Distinction between heat and temperature; definition of higher and lower temperature, and of equal temperature; the mercurial thermometer, and its graduation; Fahrenheit's, the Centigrade and Réaumur's scales. Definition of absolute zero.

7. Air is a heavy elastic fluid; the barometer; pressure of air on the sides of a vessel containing it; variations in this pressure consequent on change of volume and temperature, *i.e.* Boyle's law and Dalton's law, and the formula in which they are embodied, viz. $VP = CT$; limitation of Boyle's law; contents of the "vacuum-space" in barometers of different liquids.

8. Hydrometer of variable immersion; suction pump; force-pump; siphon; air-pump and mercurial gauge; compressed-air manometer; hydraulic press.

EXAMINATION PAPER, MAY, 1893.

Examiner: REV. J. F. TWISDEN, M.A.

GENERAL INSTRUCTIONS.

If the rules are not attended to, the paper will be cancelled.

You may take the elementary, or the advanced, or the honours paper, but you must confine yourself to one of them.

Put the number of the question before your answer.

The value attached to each question is shown in parentheses after the question. But a full and correct answer to an easy question will in all cases secure a larger number of marks than an incomplete or inexact answer to a more difficult one.

You are to confine your answers *strictly* to the questions proposed.

Your name is not given to the examiner, and you are forbidden to write to him about your answers.

The examination in this subject lasts for three hours.

FIRST STAGE, OR ELEMENTARY EXAMINATION.

INSTRUCTIONS.

You are not permitted to attempt more than *seven* questions.

1. The masses of two bodies are in the ratio of 7 to 5, and they move along straight lines under the action of forces P and Q; the velocity of the former body is increased by 12 ft. a second in 3 secs., the velocity of the latter is increased by 1260 yds. a minute in half a minute; find the accelerations of the velocities, and the ratio of P to Q. (12.)

2. Write down the equation of work and energy for a constant force acting on a particle.

A particle moving from rest is acted on through 250 ft. by a force of 9 poundals; find its kinetic energy, and, its mass being 5 lbs., find its velocity. (14.)

3. Define specific gravity.

Three pints of a liquid, whose specific gravity is 0·6, are mixed with four pints of a liquid, whose specific gravity is 0·81, and there is no contraction; find the specific gravity of the mixture. (12.)

4. State how the pressure at any point within a fluid is measured.

What error is there in the statement that the pressure at a certain point within a fluid is, for example, 90 lbs.? (10.)

5. A cubical tank has an edge 4 ft. long; it is filled with water; find the magnitude of the resultant pressure on one of the vertical sides; find also the pressure at the middle point of that side. (12.)

6. State the rule for finding completely the resultant pressure of a liquid on a body partly immersed.

A sphere is held in water with its centre in the plane of the surface of the water; find the resultant pressure of the water on it, and show in a diagram how it acts. (14.)

7. A rectangular barge, 30 ft. long, 10 ft. wide, and 4 ft. deep, weighs 20,000 lbs.; when it floats, what part of the 4 ft. will be in water? What load will be required to make it float with 3 ft. of its depth in water? (14.)

8. The specific gravity of cork being 0·25, and that of brass 8, what is the weight of a piece of brass which, when tied to a piece of cork weighing one pound, will just sink it? (14.)

9. Explain the graduation of the mercurial thermometer.

A temperature is 10° on the Fahrenheit scale; what is it on the Centigrade scale? (12.)

10. A cubical vessel, whose edge is 1 ft., is made air-tight when the barometer stands at 30 ins., and the temperature of the air is 15° C.; if the temperature of the air is raised to 60° C., what is the increase of the pressure of the contained air on each face of the cube? N.B.—The coefficient of expansion is 0·003665, and a cubic inch of mercury may be taken to weigh half a pound. (14.)

11. What is a water barometer?

A cylinder, 11 ft. long, and open at one end, is held with its open end downward in water and is just immersed; the water is kept from getting into the cylinder by forcing in more air; the water barometer stands at 33 ft.; compare the quantity of air forced in with the quantity originally in the cylinder. (14.)

12. Explain the action of the forcing-pump, and show in diagrams the valves, (*a*) when the piston is being raised, (*b*) when it is being forced down.

If the piston has a cross section of 5 sq. in. and the cistern is 40 ft. above the piston; find the number of foot-pounds of work done when the piston is forced down through 1·5 ft. (16.)

LONDON UNIVERSITY MATRICULATION.

V.—*MECHANICS.*

SYLLABUS.

Candidates will be expected to show a general acquaintance with the apparatus by which the elementary principles of physics, as enumerated below, can be illustrated and applied.

Elementary notions as to velocity, acceleration, force, mass, momentum, work and energy.

Composition and resolutions of velocities, acceleration, and forces, in one plane.

Moments and couples in one plane.

Centre of gravity, or mass centre.

Transmission of pressure in liquids; variation with depth of the pressure due to weight of liquids.

Specific gravity, and modes of determining it.

Pressure of gases, and laws relating thereto.

Atmospheric pressure.

EXAMINATION PAPER, JAN., 1894.

Examiners: PROF. J. H. POYNTING, SC.D., F.R.S., AND PROF. J. THOMPSON, SC.D., F.R.S.

1. A stone, weighing 1 ton, is suspended in the air by a chain; a rope fastened to the stone is pulled so that the chain makes 30°, and the rope 60° with the vertical. Draw a very careful figure showing the three forces acting on the stone, and a triangle representing them; find the pull on the rope.

2. A bar projects 6 ins. beyond the edge of a table, and when 2 ozs. is hung on to the projecting end, the bar just topples over; when it is pushed out so as to project 8 ins. beyond the edge, 1 oz. makes it topple over; find the weight of the bar, and the distance of its centre of gravity from the end.

3. The drum of a windlass is 4 ins. in diameter, and the power is applied to the handle 20 ins. from the axis; find the force necessary to

sustain the weight of 100 lbs., and the work done in turning the handle 10 times.

4. A weight of 2 lbs., attached to a string, falls vertically down a mine with uniform acceleration; find the value of the acceleration if the tension on the string is 1 oz. ($g = 32$.)

5. A stone, thrown vertically upwards, is observed to pass upwards through a point P, and, after an interval of 2 secs., to pass downwards through the same point; find the velocity of the stone at P.

6. Describe an experiment which proves that the mass of a body is proportional to its weight.

7. A cylinder, weighing 1 lb., floating in water with its axis vertical and each of its ends horizontal, requires a weight of 4 ozs. to be placed on its upper surface to depress it to the level of the water; find the specific gravity of the cylinder.

8. A barometer reads 30 ins. at the base of a tower, and 29·8 ins. at the top, 180 ft. above; find the average mass of a cubic foot of air in the tower, taking the specific gravity of mercury as 13·5, and the mass of a cubic foot of water as 62·4 lbs.

ANSWERS TO EXAMPLES.

CHAPTER I.

2. Unit of distance and unit of time. For time: Mean solar second. For distance: Foot or centimetre.

3. So many feet passed over in 1 sec., etc.

4. "However small the times."

5. 300. $10\frac{5}{22}$. 6. 66. 7. $16\frac{4}{11}$.

8. Reduce all to the same units of time and distance. 16, 22, 1 ft.-sec. units.

9. $6\frac{38}{45}$, $2\frac{62}{585}$.

10. The rate at which they are approaching or separating: $6\frac{1}{2}$ miles per hr., 4 hrs.

11. 1 min., $19\frac{14}{19}$ secs. (Ask yourself the question, How long will it take for the distance 10,560 ft. from van to van to be reduced to zero?)

12. Use "total distance in feet ÷ by total time." $44\frac{2206}{3113}$.

13. They will have passed each other and will be $2\frac{1}{2}$ miles apart. A is 3 miles from the first position; and B $\frac{1}{2}$ mile.

CHAPTER II.

1. Newton's first law.

2. No. Yes, if we could isolate a given mass.

3. Point of application, direction, and magnitude.

5. (*a*) Ordinary use of word weight, meaning "mass," as in "a pound of apples."

(*b*) Physical use—meaning "force."

(*c*) The interrupted attraction of the earth on the given mass.

6. (*a*) That force which acting on unit mass for unit time, etc.

(*b*) That force which acting on the mass called one pound for one second, etc.

(*c*) Independent of the intensity of gravity.

7. (*a*) The reverse of 6 (*c*).

(*b*) The pound.

(*c*) The attraction of one particle on another. In the phrase "gravitation unit," it means "influenced by the intensity of gravity."

8. 608; 5.

9. By weighing in the same locality.

10. Amount of matter in unit volume.

11. Divide mass by volume, or, practically, by the number of pounds in a cubic foot. When equal volumes however small, taken from different parts, have the same mass or weight.

12. 432; 9.

13. 11,200. No. Temperature affects the air more than the mercury.

14. By the weights of equal volumes. 49 : 26.

15. $\frac{1}{44}$ inch. 16. $12\frac{11}{72}$ lbs. 17. 153 lbs. nearly.

18. $1418\frac{3}{4}$ lbs. 19. 1 lb. to right. 20. $2\frac{1}{4}$ ins.

21. 5 and 8 in one direction; 13 in opposite.

22. -17.

23. The amount of matter in a piece of platinum called "the standard pound," preserved in London.

24. 75 : 1024, and 405 : 32. 25. 16 : 9.

26. By weighing. $(30\cdot45)^3 \times 11\cdot445$ grammes.

27. 9800.

CHAPTER III.

1. $25\cdot8$; $6\cdot5$.

2. When the angle between the forces is 90°; no; $14\cdot14$.

3. $8\sqrt{7}$ lbs.; $56\sqrt{13} \div 13$.

4. $8\cdot6$ lbs; in a direction somewhat west of N.W.

5. 90°; no; because P and Q are equivalent to R in the same direction.

6. $\sqrt{93}$ units.

8. 8 and $8\sqrt{3}$. 9. 9 and $9\sqrt{3}$.

10. 25 and $25\sqrt{3}$.

CHAPTER IV.

2. For equilibrium, either any two must be greater than the third, or two must equal the third.

3. Construct a triangle by Euclid I., 22, then draw lines from a point parallel to the sides.

6. (*a*) They must act along a straight line; the sum of two being equal to the third.

(*b*) Or along parallel lines, see Chapter VI.

(*c*) Or passing through a point and parallel to sides of some triangle taken in order.

9. $\sqrt{79}$.

10. Resultant not altered.

Chapter V.

1. Any units. Answer is a mere number.
2. 0, − 40, − 40, 0, − 40.
3. To show direction of rotation; + contrary to hands of a watch-face uppermost.

Either that the forces are in equilibrium, or that A is a point on resultant.

4. Take moments round one of the unknown forces. Then reactions are 12 and 18 lbs.
5. − 4; 20 ins. from B.
6. The moments must be equal; therefore the perpendicular on second force must equal 1; therefore second force must act at 30° to A B. Pressure on fulcrum found by considering 10 and 10 acting at 60° = $\sqrt{3}$.
7. + $8\sqrt{3}$ and $10\sqrt{3}$.

Take any point in exterior vertical angles and drop perpendiculars in the ratio of 10 to 8 on the 8 and 10 forces. Or let A C represent the 10 lbs. force and take A E on B A produced to represent the 8-lb. force. Complete the square A E F C, then the moments are equal and opposite with reference to F.

8. − 900. P = $112\frac{1}{2}$ lbs.

Chapter VI.

1. Point of application of resultant. Unchanged in relative position when all the forces are moved parallel to themselves through equal distances, or all twisted through the same angle.
3. Outside (*i.e. not* in a straight line drawn from one force to the other) and with the greater force. Between. Inversely as the forces.
4. At an infinite distance. To produce rotation.
6. 12, $3\frac{1}{2}$ ins. from 5. 2, 15 ins. beyond the 7.
7. 18 ins.
8. Decrease the 15 or increase the 10. Move point of application nearer second boy.
9. Principle of moments. 6 ft.
10. Because no point can be found round which the moments vanish.
11. One unit at a distance from the 4 lbs. equal to 3 times distance between 3 and 4.
12. 2 lbs., 10 ft. beyond the 12.
13. 14 ins. from the 5: 5 ft. beyond the 7. 14. 3 ft.
15. 10 lbs.; first support 4 lbs., second support 6 lbs.
16. 6 ins. from that end.

Chapter VIII.

3. $\frac{8}{25}$ of the length of bar from the 17.
7. That the c. g. of the ball does not coincide with the centre of ball, but is between the particular spot and the centre of ball.
9. Perpendicular from the c. g. must fall within the base.
9. Point at which all the weight may be supposed to be concentrated, half-way between that corner and the c. g. of tin triangle.

10. Neutral; stable with vertex downwards. Unstable with it upwards, *i.e.* with c. g. immediately above the tack.

11. Stable at lower end of a vertical diameter. Unstable at the upper end.

12. To lower the c. g. of vessel. Because the c. g. is lower.

13. Stable when the c. g. is vertically below the geometrical centre; unstable when above.

14. Stable on any one of its faces. Unstable on edge or corner with centre above.

15. Moment of whole round any point is equal to sum of moments of parts round same point. $26\frac{10}{13}$ ins.

16. $\frac{7}{10}$ of length of heavier from the free end.

18. $2\frac{15}{26}$ from the 4 lbs.

19. At the c. g. of triangle. Therefore c. g. of any two is half-way between them, and c. g. of these two and the third is one-third the distance, etc. At the centre of line joining c. g. of triangle with a point which divides *the* side in the ratio 2 : 1.

20. On the diagonal through the empty corner and one-fourth of diagonal from the corner.

21. Divide distance inversely as the forces. 20 ins. from the 4 lbs.

22. $\frac{1}{12}$ diagonal from centre.

23. In the common diameter 1 centimetre from centre.

24. $2\frac{1}{3}$ ft. from the thick end.

25. $\frac{1}{6}$ of diagonal from the centre.

Chapter IX.

1. Resistance overcome through a certain distance. Feet and pounds. Time.

2. Foot-pound. British absolute unit.

4. When it moves the point of application against a definite resistance. Distance multiplied by component of force in direction of motion. Product of force multiplied by effective distance. Foot-pound. Erg.

5. By the number of foot-pounds done in a unit of time.

6. 36 foot-pounds; 1152 foot-poundals; same.

7. 95 foot-pounds.

8. $196 \times 12 \times 9$ foot-pounds.

9. 12000; $\frac{5}{12}$. 10. 200. 11. $\frac{1}{15}$ horse-power.

12. $\frac{1}{11}$ horse-power. 13. 120. 14. $14\frac{1}{7}$.

15. 100 turns. 16. $89\frac{3}{5}$ gallons. 17. $1\frac{7}{33}$.

18. 8·14 mins. 19. Equal.

20. $55\frac{1}{2}$ lbs. 21. 35,840 foot-pounds. 22. 3360.

23. 165,000 foot-pounds.

24. 197,120,000; 28 hrs. 26 mins., 40 secs.

25. 6000; $\frac{1}{3}$. 26. 264,000. 27. 1650; 20.

Chapter X.

1. By using the word "increase" in the algebraical sense of positive or negative. A force.

2. Second and foot. Because, if the formula be applied to falling bodies, the moving body may again return to the starting-point, and then $s = 0$. s is the distance from the starting-point, at the end of time t.

3. (*a*) 24 ft. above starting-point.
(*b*) Body has returned to starting-point.
(*c*) Body has gone "up," back again through the starting-point, and is now 20 ft. below it.

4. (*a*) The acceleration is negative and the initial velocity has gradually been destroyed.
(*b*) See (*c*) in Answer 3.

5. A rectangle. A right-angled triangle, base 3, and perpendicular 21.
6. 6000. 7. $\frac{3}{8}$. 8. 1920.
9. 10. 10. 55; 66,000. 11. 24000. 12. 50
13. 3 secs.; 5 secs.; ball rolling along ground.
14. Yes; a constant force.
15. Its numerical value in given units, and its direction.
16. 30 miles per hour.
17. That if the velocity were, say 10 ft. per second at the commencement of a certain second, it would be 15 at the commencement of the next, 20 at the next, etc.
18. By the number of units of velocity added in unit time. $\frac{3}{10}$ ft. per second. 65 ft.
19. 250 ft. $2\frac{1}{2}$, $7\frac{1}{2}$, $12\frac{1}{2}$, $32\frac{1}{2}$.
20. See last example. The common difference is 5 ft.
21. 25 ft. per second. 2 seconds.
22. $15\frac{3}{4}$. 23. $1928\frac{4}{7}$. 24. 19, 115 ft.
25. $\frac{1}{3}$ secs., 0·138 secs., 0·106 secs. 26. 32 27. 20.
28. $f = 8$ and $V = 5$. 29. 245.
30. $v = ft$ and $s = \frac{1}{2} ft^2$.
31. $\sqrt{\dfrac{99 \text{ ft.}}{1320}}$ secs.
32. $s = vt + \frac{1}{2}ft^2$; 7.
33. 20. 34. 15. 35. $78\frac{3}{4}$.
36. Any units—generally pounds and feet per second. No name for the "unit of momentum."
37. 64 : 9. 38. 7 : 8. 39. 33 : 20.
40. P = acting force in poundals.
m = mass of body in pounds.
f = its acceleration in feet per second.

Now $V = ft$, *i.e.* $f = \dfrac{v}{t}$ Substituting therefore

$$P = m\frac{v}{t} \text{ i.e. } Pt = mv.$$

41. Poundals. 80 poundals.
42. P = 20 poundals. Q = 21 poundals. Required ratio: $\frac{4}{21}$.

Chapter XI.

1. v is initial velocity; u is final velocity both in feet per second. m is weight of body in pounds. R is resistance in poundals; s is the distance passed over in feet. Kinetic energy.

2. Ability to do work in virtue of body's position, *e.g.* a weight wound up to top of pile-driving machine. Ditto in virtue of its velocity. The two add up to same number. Energy can never be created nor destroyed.

There is the same amount of energy in the universe to-day as when it was created.

3. (*a*) When m is taken as pounds
   (*b*) When m is taken as $\frac{\text{lbs.}}{32}$
4. (*a*) 25,600 and 800
   (*b*) 51,200 and 1600
5. 6250 and 6050. 6. $5\sqrt{3}$. 7. 1372 ft.
8. $1\frac{11}{81}$ ft. 9. 24,000 foot-pounds. 160 foot-seconds.
10. 14,400 foot-poundals and 450 foot-pounds.
11. $390\frac{5}{8}$ foot-pounds, $781\frac{1}{4}$ ft.
12. 192 foot-poundals. 5 ft. per second.
13. 1080; kinetic energy. 14. $\frac{25}{81}$; $44\frac{4}{5}$.
15. 1600 and 800. 16. 720 ft.

Chapter XIII.

1. Solid, liquid, gaseous. Water.
3. The absence of friction between the component particles.
4. A fluid in which friction is entirely absent, both among its own particles, and between the surfaces in contact with it, and itself. No. The tea, after being set whirling, gradually comes to rest—the liquid at the edges of the cup first.
5. Because friction is absent, and hence there can be no component along the surfaces.
7. Treacle, oil; viscosity.
8. Pressure applied to one part of a fluid at rest is transmitted equally in all directions.
9. Piece of sheet tin introduced vertically into a vessel of water, etc.
10. Because the force is applied to the water tangentially and there is thus no very great resistance. Because the force is now applied non-tangentially, and the inertia of the water becomes very great.
12. No meaning. Pressure on a unit of area containing the point.
13. See text for proof. No; it is curved.
14. (*a*) It is the same in all directions.
    (*b*) It varies with the depth below the surface.
    (*c*) It depends also upon the nature of the fluid.
15. This follows from Pascal's law.
16. Ditto.
17. Pascal's law. The unequal plungers. The water connections and the leather collar.
18. $16\frac{1}{14}$ lbs. 19. 3600 lbs. 20. 400 lbs.
21. 60 lbs.
22. Pressure on the smaller piston is found from the proportion $72^2 : 2^2 = 14 \times 2240 : x$.

$$i.e.\ x = \frac{7 \times 280}{81}\ \text{lbs.}$$

Power required is obtained by taking moments round the fulcrum—

$$i.e.\ 24 \cdot \mathrm{P} = \frac{3 \times 7 \times 280}{81}$$

$$i.e.\ \mathrm{P} = 3\tfrac{2}{81}\ \text{lbs.}$$

23. $5\frac{1}{3}$ lbs.

CHAPTER XIV.

1. The depth below the surface and the nature of the liquid. Pressure in pounds is equal to area of surface immersed (in feet) × depth of c. g. (in feet) × sp. gr. × $\frac{1000}{16}$.

2. 250 lbs. Lid, nothing; bottom, 500 lbs.

3. $\frac{1000}{72}$ lbs.; $\frac{1250}{72}$ lbs. 4. $\frac{1000}{144}$ lbs.

5. The sum of all the pressures acting on the surface or surfaces. 18,000 lbs.

6. 20,000 lbs. 8. $\frac{1000}{768}$ lbs.

9. $(\frac{22}{7} \times 1^2 \times \frac{5}{2} \times \frac{1000}{16} + \frac{22}{7} \times 1^2 \times \frac{5}{2} \times \frac{915}{16})$ lbs.

10. Pressure without the lid is $1767\frac{6}{7}$ lbs. The pressure produced by the lid is 3 lbs. on every πr^2 sq. ft. of surface; *i.e.* 18 lbs.; therefore total answer $1785\frac{6}{7}$.

11. It produces 4 lbs. pressure on each area the size of the lid, $1553\frac{3}{14}$ lbs.

12. Proportional to the depth below the surface. Hence the required ratio is $1\frac{1}{2} : 3\frac{1}{2}$; *i.e.* $3 : 7$.

$$\text{Resultant pressure} = \pi r^2 \times 2\tfrac{1}{2} \times \tfrac{1000}{16} \text{ lbs.}$$
$$= 491\tfrac{1}{14} \text{ lbs.}$$

13. 140,625 lbs.: 62,500 lbs.

14. Multiply area immersed (in square feet) by depth of c.g. (in feet) by 64 lbs. by specific gravity of fluid $\frac{22}{7} \times 3^2 \times 3 \times \frac{1000}{16}$ lbs.

15. 250 lbs. 16. 2021·04 grains.

17. $1687\frac{1}{2}$ lbs. on base, $843\frac{3}{4}$ on sides; 3105 and $1552\frac{1}{2}$

18. 500 lbs.; 1000.

19. $334\frac{29}{144} : 2864\frac{7}{12}$; $39 : 668\frac{29}{72}$.

20. 6000 lbs.; 2000 lbs. The pressures on the sloping sides act perpendicularly to the sides, and hence give a component vertically downwards on to the base.

21. $34\frac{13}{18}$ lbs. 22. $5\frac{5}{24}$ lbs.

23. Depth of A : depth of B.

Let x be depth of A below surface in feet, and y ditto for B.

$$\text{Then } x = 47$$
$$\text{And } x - 1 = 9(y - 1). \quad \text{Whence } x = 6\tfrac{2}{5},\ y = 1\tfrac{3}{5}.$$

24. Not always. The whole pressure is the sum of the various pressures. The resultant pressure is found by combining the various pressures by the parallelogram law, or laws for parallel forces. See text.

25. Equal to the weight of the fluid displaced. Principle of Archimedes.

26. See text.

27. See Answer 2. (1) Volume immersed $\frac{1}{3}$ cub. ft., therefore pressure is $20\frac{5}{6}$ lbs.

(2) Volume immersed is $\frac{1}{6}$ cub. ft., therefore pressure is $10\frac{5}{12}$ lbs.

Join middle point to AE to C. Take K $\frac{1}{3}$ of this line from AE. Draw line parallel to edge of cube (*i.e.* at right angles to the paper) through K. The resultant fluid pressure acts vertically upwards through the middle point of this line.

28. Tension in string plus weight of cork in air is equal to weight of water displaced.

29. $7\frac{11}{27} - 2\frac{17}{54} = 5\frac{5}{54}$ lbs.

30. (*a*) Pressure on a unit of area containing the point.

(*b*) Whole pressure on the surface divided by area of surface.

(*c*) Sum of pressures on the various surfaces under consideration.

(*d*) The pressures on the various surfaces under consideration compounded by the parallelogram law, or laws for parallel forces.

(*e*) Point of application of the resultant pressure on any surface immersed.

31. (*a*) $\frac{2}{3}$ of the height downwards.

(*b*) $\frac{1}{2}$ way down.

(*c*) $\frac{3}{4}$ way down.

32. 1500 lbs. Thus: the resultant pressure is 4500 lbs. acting 4 ins. from C D. Take moments round C D.

$$i.e.\ 12\,P = 4 \times 4500$$
$$\therefore P = 1500$$

33. 16,000 and 20,000 lbs.

34. Tension + weight of water displaced = weight of body.

$$i.e.\ T + 5000 \text{ ozs.} = 6000 \text{ ozs}$$
$$i.e.\ T = 1000 \text{ ozs.}$$

35. Area exposed = 50×13 square feet.

$\therefore$ whole pressure = $50 \times 13 \times 6 \times 1000$ ozs.
= 243,750 lbs.

36. $\frac{125}{3}$ lbs. and $\frac{250}{3}$ lbs.

Chapter XV.

1. (*a*) Centre of gravity and centre of buoyancy must be in a vertical line.

(*b*) Weight of body must equal weight of displaced liquid.

2. The centre of gravity of the displaced fluid.

(*a*) C of G below the C of B

(*b*) C of G above the C of B.

No; we need to know the position of metacentre.

3. The point in which the vertical through the centre of buoyancy of a floating body which has undergone a slight displacement intersects the line drawn vertically through the centre of buoyancy in original position of equilibrium. Because the centre of gravity is likely to be below the centre of buoyancy, and hence the acting couple has the tendency mentioned.

4. Metacentre above the centre of gravity.

5. $\frac{3}{2} .\ a .\ 600$ ozs. $= x .\ a .\ 1000$ ozs.

where a = cross-section of wood in square feet.

whence $x = \frac{9}{10}$ of a foot.

6. Let x be the length of edge in inches floating in oil, then the equation is obvious—

$$\frac{216}{1728} \times 970 \text{ ozs.} = \frac{(6 - x)\,36}{1728} \times 1028 \text{ ozs.} + \frac{x \times 36}{1728} \times 915.$$

Whence $x = 3\frac{9}{113}$ ins.

7. $\frac{3}{4}$; $140\frac{5}{8}$ lbs. 8. 1·4; $147\frac{76}{224}$ lbs. 9. 1728.

10. The specific gravity of fluid must be $\frac{4}{3}$ the specific gravity of the body. Hence the same volume of fluid must weigh 4 lbs. Hence $\frac{1}{4}$ of the volume will weigh 1 lb., and this is the pressure on the hand

Chapter XVI.

2. (*a*) Whatever be the weight of a volume of water, the weight of same volume of oak is $\frac{7}{10}$ of that weight: (*b*) water, (*c*) air.

3. Weight (in lbs.) = volume in cubic feet $\times$ specific gravity $\times \frac{1000}{16}$.

4. (*a*) $61\frac{29}{36}$ lbs.; (*b*) $46\frac{7}{8}$ lbs.
5. $\frac{3}{16}$ (lbs.) $= 18 \times a \times 8{\cdot}8 \times \frac{252}{7000}$ lbs., whence $a = 0{\cdot}03$ square inch.
6. 8 lbs. $9\frac{1}{2}$ ozs.
7. 30 ozs. $= 2Q \times 0{\cdot}916 \times 20$, whence $Q = 0{\cdot}83$.
8. $38\frac{1}{2}$ lbs. $= G \times 0{\cdot}88 \times \frac{160}{16}$ lbs., whence $G = 4\frac{3}{8}$ galls.
9. 3 lbs. $= 20 \times$ specific gravity $\times \frac{1}{23}$ lbs.; whence specific gravity $= 4.2$.

10. $7{\cdot}2 = \dfrac{4{\cdot}112}{\text{wt. of water disp.}}$ weight of water displaced is $\frac{560}{9}$ lbs. The actual weight displaced is 80 lbs. Therefore $\frac{160}{9}$ lbs. is too much, *i.e.* the body is hollow, and the cavities the number of cubic feet in $\frac{160}{9}$ lbs. of water. Whence volume of cavities is to volume of solid as 2 : 7, therefore apparent volume being 9, the cavities are $\frac{2}{9}$ of this apparent volume.

11. Volumes are proportional to cubes of radii, therefore ratio required $= \dfrac{1^3 \times 7}{2^3 \times 2}$ *i.e.* $\frac{7}{16}$.

12. $0{\cdot}95 = \dfrac{\text{weight of oil}}{\text{weight of same vol. of water}}$

$\therefore$ Answer $0{\cdot}95 \times 27$ ozs. $= 25{\cdot}65$ ozs.

13. 388·8 lbs.

14. Specific gravity $= \dfrac{\text{wt. of body}}{\text{wt. of same vol. of water}}$ (Principle of Archimedes.)

15. $5\frac{9}{11}$. 16. 20·86. 17. 5·22.
18. $2\frac{1}{7}$. 19. 20.

20. Specific gravity $= \dfrac{732}{732 - 252} = \frac{730}{480} = 1{\cdot}525$.

21. Glass bottle, well ground stopper with hole through it, carefully wipe, avoid holding in warm hand, keep temperature the same during the experiment.

22. 0·786. 23. 0·758.

24. Wt. of water displaced $47 - 22 = 25$ grs.
„ alcohol „ $47 - 25{\cdot}8 = 21{\cdot}2$ grs.

$\therefore$ Specific gravity $= \dfrac{21{\cdot}2}{25}$ (Principle of Archimedes.)

25. Specific gravity $= \dfrac{\text{wt. of acid}}{\text{wt. of water}} = \dfrac{20{\cdot}8 - 19{\cdot}36}{20.8 - 19{\cdot}86} = \frac{144}{91} = 1.53$.

26. $2{\cdot}7 = \dfrac{3}{\text{wt. of water}}$ $\therefore$ specific gravity required $= \frac{27}{30} = 0{\cdot}9$.

27. Weight of first liquid displaced = 3 lbs.
„ same volume of second liquid = 2 lbs.
$\therefore$ specific gravities are 3 : 2.

28. $1\frac{7}{61}$.

29. $0{\cdot}9 = \dfrac{\text{wt. of alcohol}}{\text{wt. of equal vol. of water}} = \dfrac{\text{wt. of alcohol.}}{35 - 21}$ $\therefore$ Wt. in alcohol $= 35 - 12{\cdot}6 = 22{\cdot}4$ grs.

30. $2\frac{1}{7}$ and $\frac{5}{7}$. 31. $\frac{35}{37}$.

32. First weight of compound body in water = weight of sinker in water + weight of body in water.

$\therefore$ weight of body in water = weight of comp. body − weight of sinker. *i.e.* $Bw = Cw - Sw$.

$\therefore$ specific gravity $= \dfrac{B\,air}{B\,air - (Cw - Sw)}$.

33. $7 \cdot 8 = \dfrac{32 \text{ ozs.}}{\text{wt. of disp. water}}$ $\therefore$ wt. of brass in water $= 32 - \dfrac{32}{7 \cdot 8}$

$\therefore 25 \cdot 78 = 32 \left(1 - \dfrac{1}{7 \cdot 8}\right) +$ wt. of cork in water. Whence specific gravity $= \dfrac{177}{199 \cdot 53} = 0 \cdot 58 \ldots$

34. $300 = 380 +$ wt. of mahogany in water

$\therefore$ wt. of mahogany in water $= 80$

$$\therefore \text{specific gravity} = \frac{375}{375 - \text{wt. of mahog. in water}} = \frac{375}{375 - (-80)} = \tfrac{375}{455} = \tfrac{75}{91}$$

35. $\tfrac{1362}{2731}$. Thus $8 = 22 - \dfrac{22}{11 \cdot 35} +$ wt of wood in water. From this wt. of wood in water $= -\tfrac{2738}{227}$.

$$\therefore \text{specific gravity} = \frac{12}{12 + \frac{2738}{227}} = \tfrac{2724}{5462} = \tfrac{1362}{2731}.$$

36. $\tfrac{309}{517}$.

37. Specific gravity $= \dfrac{\text{wt. of body in air}}{\text{wt. of disp. water}} = \dfrac{\text{wt. of body in air}}{\text{wt. of disp. liquid}} \times \dfrac{\text{wt. of disp. liq.}}{\text{wt. of disp. water}} = \dfrac{\text{wt. of body in air}}{\text{wt. of disp. liquid}} \times$ specific gravity of liquid.

$$\text{Specific gravity} = \frac{29 \cdot 5}{1} \times 86 = 25 \cdot 37.$$

38. Specific gravity $= \dfrac{1 \cdot 23}{1 \cdot 27} = 0 \cdot 96$.

39. Specific gravity $\times 15 = 3 \times 1 \cdot 2 + 5 \times 0 \cdot 08 + 7 \times 0 \cdot 94 -$ specific gravity $= 0 \cdot 7$, etc.

40. $(x + 1)\ 1 \cdot 015 = 1 \cdot 02 + x$; $\therefore x$, *i.e.* the quantity of water added to 1 volume of milk $= \tfrac{1}{3}$.

41. $12 \times$ specific gravity $= 11 \times 19 \cdot 4 + 1 \times 8 \cdot 8$. Answer $18 \cdot 516$.

42. Specific gravity of whole $\times$ cont. volume $=$ sum of weights of parts.
i.e. $V\,(1 \cdot 6) = 3 + 7 \times 1 \cdot 84$ whence the contracted volume is $9 \cdot 925$
$\therefore$ 10 has contracted by $0 \cdot 075$ of a volume
$\therefore$ 100 ,, ,, ,, $0 \cdot 75$
$\therefore$ Answer $= \tfrac{3}{4}$ per cent.

43. Specific gravity $\times \left(\tfrac{3}{2}\right) = 15 + \tfrac{1}{2} \times 12 = 21$.
$\therefore$ Specific gravity $= 14$

44. $(2 \cdot 5)(V_1 + V_2) = V_1\ 1 \cdot 5 + V_2\ (3)$.
$$\frac{w_1 + w_2}{2 \cdot 5} = \frac{w_1}{1 \cdot 5} + \frac{w_2}{3}$$

45. $(x + 1)(1 \cdot 009) = x + 1 \cdot 027$, whence $x = 2$ parts of fresh water to 1 of salt water.

46. $S\,(V_1 + V_2) = S_1 V_1 + S_2 V_2$; $1 \cdot 175$.

47. By subtracting the remaining weights from standard weight when the body is on pan and the instrument sinks to level.

48. $2 \cdot 5$.

49. Weight of displaced liquid in first case is 10; ditto in second is 13. $\therefore$ specific gravity $= \tfrac{10}{13}$.

50. x in lower pan $= 2$ in upper; *i.e.* $x - \dfrac{x}{8} = 1$; $\therefore x = 2\tfrac{2}{7}$.

51. $1\tfrac{6}{13}$. 52. 8. 53. $0 \cdot 8$.

54. 4 lbs. 55. $7\frac{3}{5}$. 56. Weight of water displaced ; 75 lbs.
57. $62\frac{1}{2}$ lbs. 58. 12 and 6.
60. $\frac{5}{6}$. 61. 2080. 62. 49 : 26.
63. $\frac{1}{44}$ inch.

64. The displaced air will weigh $300 \times 0{\cdot}31$, *i.e.* 93 grs. Hence the bladder + the hydrogen must weigh 93 grs. The hydrogen weighs $300 \times 0{\cdot}02$, *i.e.* 6 grs ; therefore bladder will weigh 87 grs.

65. Heights above the common level are inversely as the specific gravities. $12\frac{1}{2}$.

66. $4\frac{1}{3}$.

67. Wt. of gold − wt. of displaced nitric = wt. of silver − wt. of displaced alcohol.

$$\text{Now } 19{\cdot}3 = \frac{\text{wt. of gold}}{\text{wt. of nitric}} \cdot \frac{\text{wt. of nitric}}{\text{wt. of water}}$$
$$= \frac{Wg}{\text{wt. of nitric}} \times 1{\cdot}5$$
$$\therefore \text{wt. of nitric} = \frac{Wg \times 1{\cdot}5}{19{\cdot}3} \text{ etc.}$$
$$\text{whence } Wg : Ws = 37{,}380 : 37{,}249.$$

68. 9·06. $\text{Thus } x = \dfrac{\text{wt. of cylinder}}{\text{wt. of disp. mercury}} \times 13{\cdot}6$
$$= \frac{W}{\frac{3}{2}W} \times 13{\cdot}6 = 9{\cdot}06.$$

69. $13\frac{8}{9}$ ozs. 70. 0·25.

Chapter XVIII.

1. See Text. No. The vessel in which the air is weighed should not change its shape.

2. $1\frac{1}{4}$ oz: $\dfrac{1\frac{1}{4}}{1000} = \dfrac{5}{4000} = \dfrac{1}{800}$

3. A fluid, the particles of which exert a constant pressure on the sides of containing vessel. Yes.

5. No. The point arrived at, at which, if the pressure be slightly increased, the liquid state is assumed: if slightly decreased, the gaseous is resumed.

6. For gases liquified with difficulty—oxygen, air, etc. For those easily liquified—*e.g.* carbonic acid, etc.

7. This is Dalton's law, and is another way of stating Boyle's law. "Each gas produces the same pressure that it would if the others were not present."

8. No. "When a gas is submitted to pressure, the volume is inversely proportional to the pressure produced."

10. $1 \times 33 = x\,(44 + 33)$. $\therefore x = \frac{3}{7}$ foot.

11. Pressure due to 14 ft. of water = 14 × 1000 ozs. = 875 lbs.

12. The water would fall because there is nothing now to counterbalance the weight of the 20 feet of water.

13. $15 \times 12 = \dfrac{12 + 4}{3}x$. $\therefore \times x = 33\frac{3}{4}$ lbs. per square inch.

14. Pressure of 4 ins. $\times (x + 4) \times \frac{1}{4}$ sq. in. $= \frac{1}{4}$ cub. in. $\times$ 30 ins.
$\therefore x = \frac{7}{2}$ lin. ins.
$\therefore$ required volume $= \frac{7}{2} \times \frac{1}{4} = \frac{7}{8}$ cub. ins.

16. $1 \times 30 = (x + 4)\,4$. $\therefore$ vol. is 3·5 cub. ins.
17. $7\frac{1}{3}$ cub. ins.
18. Original pressure $= 3 \times 15 = 45$ lbs. Then $4{\cdot}45 = x\ (54)$. $\therefore x = 3\frac{1}{3}$: whence the piston has descended 8 ins. In second case it has descended $1\frac{9}{13}$ ft.
19. Suppose it falls x ins. Then we have the equation $2 \times 30 = (10 + x) \times \frac{1}{3}x$. Whence $x = 10$ ins.
20. Suppose x cub. ins. Then $x \times 30 = (10 + 4) \times \frac{1}{5} \times 4$. Whence $x = \frac{28}{75}$ cub. ins.
21. 1 in.
22. Pressure is due to $32 + 85 = 117$ ft. of sea water = 15 lbs. $13\frac{1}{2}$ ozs.
23. 30 ft.
24. The difference of the barometer readings = 10 ins. This corresponds with $13{\cdot}6 \times 10 = 136$ ins. $= 11\frac{1}{3}$ ft. of water. Therefore top of bell is 5 ft. 4 ins. below water.
25. 320 cub. ft.

Chapter XIX.

1. The condition of a body with regard to the amount of heat it possesses capable of affecting our senses. No.
2. Temperature is related to heat in the same way that the surface of the water is related to the water itself. No.
3. When one receives from the other the same amount of heat that it gives to the second in proportion to its size. No—not necessarily.
4. That A gives more heat to B than B gives to A.
5. No. Because our body would give off more heat than it receives, and hence our temperature would gradually fall.
6. Heat.
7. Yes. Become latent, *i.e.* "hidden" as far as a thermometer is concerned; it is doing work in driving the particles of the body further asunder than in the previous condition.
9. To indicate the temperature of a room, etc.
10. (1) Does not wet its glass envelope. (2) Runs through a great range of temperature without freezing or boiling. (3) Quickly transmits heat throughout its substance. (4) Does not require much heat to raise its temperature.
11. The scale designed by the philosopher Fahrenheit, used commonly in England; that designed by Celsius, and used in France and for all scientific purposes; ditto by Réaumur, in Russia and Sweden.
12. 32°; 0°; 0°; 212°; 100°; 80°.
13. No; alteration of pressure affects them.
14. Subtract 32° and multiply by $\frac{5}{9}$; multiply by $\frac{9}{5}$; and add 32°.
15. $(F. - 32)\,\frac{5}{9} = C. = \frac{5}{4}\,R.$; 100°; 15; $-6\frac{2}{3}$; 100; $112\frac{7}{9}$.
16. 13° below zero, *i.e.* $32 + 13\ (45^\circ)$ below freezing point -15; $-32\frac{2}{9}$; -15°; -50.
17. $63\frac{1}{2}$; 5, $49\frac{7}{9}$; 356.
18. $200 \quad -8 \quad -32$; -32°.
19. $C = \frac{5}{9}\,(F. - 32) = \frac{5}{9}\,(-40 - 32) = \frac{5}{9}\,(-72) = -40^\circ$.
20. See Answers 3 and 8.
26. The "boiling point" is the point at which the mercury (*e.g.*) stands when the instrument is held in the steam of pure boiling water at the ordinary pressure of 30 ins. of mercury. It may be considered fixed, if the conditions of pressure do not alter.

CHAPTER XX.

(1) $\frac{1}{273}$; $\frac{5}{9}$ of $\frac{1}{273} = \frac{1}{491}$ nearly.

(2) Dalton and Gay Lussac's law; if Vt be the volume of a gas at the temperature $t°$ C., and Vt its volume at T° C.; then $\frac{Vt}{VT} = \frac{273 + t°}{273 + T°}$.

Charles's law; the pressure remaining the same, the volume of a gas is directly proportional to its absolute temperature.

3 and 4. V = volume of gas at temperature t and pressure p.
V = volume of gas at temperature and pressure P. α is the co-efficient of expansion, namely $\frac{1}{273}$.

The properties of gases embodied are extension, compressibility, elasticity, and weight.

5. $107\frac{2}{3}$.

6. $\frac{V}{500} = \frac{273 - 20}{273 + 20}$ $\therefore V = \frac{500 \times 253}{293}$

7. 64 cub. ins. nearly.

8. 80 cub. ins. nearly.

9. 3·8 cub. ft.

10. 15·34 cc. 11. 0·56 nearly.

12. A volume of gas contracts $\frac{1}{273}$ part for each C. degree of cooling; therefore 273° C. below zero the volume would be 0. This temperature 273° C. below zero is called the absolute zero.

13. $273 + 14 = 283°$ C.

14. $273 - 14 = 259°$ C.

15. 459° F. $459 + 32 = 491$.
$459 + 16 = 475$.
$459 - 40 = 419$.

CHAPTER XXI.

1. The varying pressure of air on a square inch, say. Rises in barometer with increased pressure, in thermometer with increased temperature, etc.

2. The downward pressure of a column of air equal in weight to that of the column of mercury.

3. Because the pressure is the same at all points of a horizontal plane in a fluid.

4. Pressure diminishes—less air above—ascending currents, etc.

5. For small variations in height, the column falls $\frac{1}{10}$ of an inch for 87 ft. of ascent. Or let H_t and H_b be the height of barometer column at top and bottom of mountain: height required is $\frac{H_b - H_t}{H_b + H_t} \times 50{,}428$ ft.

6. The pressure at a point when the barometric column is 30 ins. high. One atmosphere is the pressure of about 30 ins. of mercury or $14\frac{3}{4}$ lbs. to square inch.

7. Because air being moist its specific gravity is diminished and the pressure is less, etc.

9. Yes. Must be held vertically. If held obliquely, its true height is found by running up a vertical, and drawing a horizontal line to it from top of column.

11. The height of a column of air of the same density throughout as it is at the earth's surface. Constant if taken as a column equal in weight to 30 ins. of mercury.

Mercury column $= \frac{30}{1728} \times 13{\cdot}6 \times \frac{1000}{16}$ lbs.

$$\text{Air column} = \frac{x}{1728} \times \frac{1}{800} \times \frac{1000}{16} \text{ lbs.}$$

Where x = height of homogeneous atmosphere.

Whence $x = 27{,}200$ ft.

12. See text. Force acting along piston rod is equal to weight of a column of water having a base equal in area to that of piston, and height equal to difference of heights of water in cistern and pump. When the pump is in full action and discharging a volume of water equal to the volume of the barrel at each stroke, the tension is of the piston rod constant, and is equal to weight of water in a column of height distance between water in cistern and pump spout, and base as above.

13. Tension in rod $= 24 \times 20$ lbs. Distance moved through at each stroke $= \frac{1}{3}$ ft. Foot lbs. of work $= 24 \times 20 \times \frac{1}{3} = 160$.

14. Tension $= \frac{8}{144} \times 12 \times \frac{1000}{16}$ lbs. $= 41\frac{2}{3}$ lbs.

15. Because the atmospheric pressure (which supports the column of water in the pipe) is incapable of sustaining a column of water greater than 34 ft. Use the force pump or lifting pump.

16. 30·4 ft. nearly.

17. 1 lb. $15\frac{1}{4}$ ozs.

18. 380·8 to 421·6 ins.

19. See text for explanations. Force required $\frac{22}{7} \cdot 4 \cdot \frac{1}{144} \cdot 20 \cdot 64$ lbs. $= 111\frac{47}{63}$ lb.

20. $50 \times \frac{8}{144} \times \frac{1000}{16}$ lbs. $= 173\frac{11}{18}$ lbs.

21. Because a partial vacuum is created which must be filled by something, viz. the water. $138\frac{8}{9}$ lbs.

23. Because the pressure at the bend will be greater when there is a greater difference between the legs—provided the siphon be not too high.

24. Practically no effect—if siphon be not too high.

25. The liquid in the short tube will flow back to the vessel, and that in longer will flow out at other end until the instrument be empty.

26. (1) If in longer leg below the level of short end no effect.

(2) If no longer leg above the level, all liquid below the hole will continue to flow downwards. That above the hole will ascend and flow down the shorter leg and the siphon will be empty.

(3) If in shorter leg, all below will flow back; all above will ascend and flow down the long leg till siphon be empty.

27. The siphon would continue to work until the water barometer be equal to the height of the water in shorter leg. After that it will cease to work, the liquid flowing downwards through both legs till siphon be empty.

28. Yes, from the less dense to the denser.

29. 29 ins. about.

30. Just less than $13\frac{3}{8}$ feet.

31. See description in the text.

32. The chief advantage is that the air can be exhausted much more quickly, owing to the double barrel; also the atmospheric pressure resists the ascent of one, but accelerates the descent of the other piston. Hence its effect is neutralized and thus the pump is easily worked.

33. See text for the formula. Density at commencement : that at end as 1 : 0·192.

34. 1 to 0·43 nearly. The valves not being exactly air tight and the pressure of air in receiver being eventually insufficient to force open the valve into receiver.

35. $\frac{1}{8}$.

37. The volume required for the original air under a pressure of 27 is found by Boyle's law to be $\frac{30}{27}$ of the original. But the only available volume for it is $\frac{27}{27}$; therefore $\frac{3}{27}$ or $\frac{1}{9}$ of original volume has been withdrawn.

38. 25·2.

39. $\frac{15}{x} - (\frac{6}{5})^3$, whence $x = 8\frac{49}{72}$ lbs.

40. $\frac{30}{33} = (\frac{10}{11})^n$. Whence $n = 1$, *i.e.* after one descent.

42. 13 ft. $6\frac{54}{85}$ ins. 43. 40 lbs.

44. 2 ins. from top. Difference of level 12 ins.

Science and Art, 1893.

1. Accelerations are 4 ft. per second in every second and 2·1 ft.

$$P : Q :: 8 : 3$$

2. V = 30 ft. per second. Kinetic energy = 2250 foot-poundals.

3.
$$3 \times 0·6 + 4 \times 0·81 = 7 \times S$$
$$1·8 + 3·24 = 7 \times S$$
$$5·04 = 7\ S. \quad \text{Ans. } s = 0·72$$

4. The statement in the second part of the question should have been "at the rate of 90 lbs. per unit of area."

5. Area = 16 sq. ft.

Depth of c.g. = 2 ft.

Pressure $= 16 \times 2 \times \frac{1000}{16}$ lbs. = 2000 lbs. answer. Pressure at middle point of side $= 2 \times \frac{1000}{16}$ lbs. = 125 lbs. Answer, 125 lbs. per square foot.

6. Remember that the pressure of the liquid equals the weight of the fluid displaced, and acts vertically upward through the centre of gravity of the displaced liquid.

7. Weight of barge = 20,000

Let x = depth under water

then vol. immersed $= 30 \times 10 \times x$

$\therefore$ wt. of water disp. $= 30 \times 10 \times x \times \frac{1000}{16}$ lbs.

$\therefore 30 \times 10 \times x \times 62·5 = 20{,}000$

$$x = \frac{20{,}000}{10 \times 30 \times 62·5} = \frac{200}{187·5} = \frac{16}{15} = 1\tfrac{1}{15}. \quad \text{Ans. } 1\tfrac{1}{15}.$$

Load required to make it float with 3 ft. of its depth in water

$$30 \times 10 \times 3 \times 62·5 = \text{weight of displaced water}$$
$$56{,}250 \text{ lbs.} = \text{weight ,, ,, ,,}$$
$$\therefore \text{weight required} = (56{,}250 - 20{,}000) \text{ lbs.}$$
$$= 36{,}250 \text{ lbs.}$$

8. $1 + w = (V + V^1)\, w$

$$1 + w = \left(\frac{1}{·25 \times 62·5} + \frac{w}{8 \times 62·5} \right) \times 62·5$$

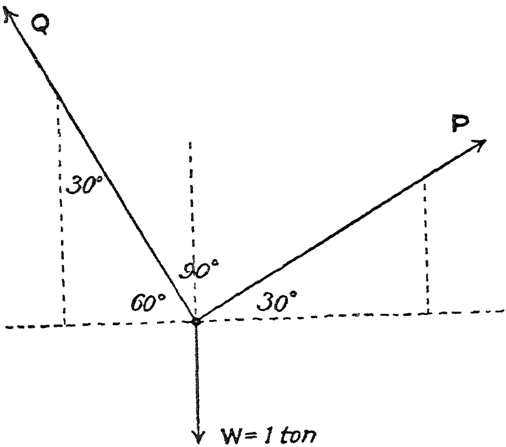
$$\text{or } 1 + w = 4 + \frac{w}{8}$$

$$3 = \frac{7w}{8}\ ;\ w = \frac{24}{7} \text{ lbs.} = 3\tfrac{3}{7} \text{ lbs.}$$

9. $-12\frac{2}{9}°$ C. In defining the boiling point, note that it is the temperature of steam *under a standard pressure.*

10. 337·5. 11. $\frac{1}{3}$ original volume. 12. 1·30 $\frac{5}{24}$ foot-pounds.

LONDON MATRICULATION. (JAN., 1894.)



2. Weight of bar = 2 ozs. Distance of c. g. from end = 12 ins.

3 400π foot-pounds.

4. Acceleration = 31 ft. per second in every second.

5. Time to highest point = 1 sec. Velocity through P = 32 ft. per second.

7. 0·8.

8. A column of air 180 ft. long makes a difference in barometer of 0·2 in.

Or 1 ft. of air $= \frac{0·2}{180}$ in. of mercury $= \frac{1}{900}$ in. of mercury. ∴ $\frac{1}{900}$ in. of mercury over 1 sq. ft. has a weight of

$$\frac{144}{900} \times \frac{62·4}{1728} \times 1·35 \text{ lbs.}$$
$$= \frac{16}{100} \times \frac{62·4 \times 13·5}{1728} \text{ lbs.}$$
$$= \frac{62·4 \times 13·5}{100 \times 108} \text{ lbs.} = \frac{62·4 \times 1·5}{100 \times 12} \text{ lbs.}$$
$$= \frac{5·2 \times 1·5}{100} \text{ lbs.}$$
$$= 0·078 \text{ lbs.}$$

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